COMP1600/COMP6260: Mid-semester exam (Common Mistakes)

2021-2

Q2: Propositional Logic + Natural Deduction

1. Proving the rule

   $$\frac{a \to (b \to c)}{a \wedge b \to c}$$

   instead of the axiom

   $$(a \to (b \to c)) \to (a \wedge b \to c)$$

2. Not stating that the formula is a tautology.

3. Mistakes with the indent/scoping of both implication introduction and implication elimination.

   That is, not creating a new indent for an assumption towards imp intro, closing off 0 or 2 indent levels when using imp elim.

Q3: First Order Logic + Natural Deduction

1. Assuming the opposite of what was given and then continuing the proof under the false assumption.

1	$\neg \forall x. \exists y. \neg P(x, y)$	
2		$\neg (\neg \forall x. \exists y. \neg P(x, y))$
3		

2. Applying rules to the content inside the ¬ . E.g., for all elimination done disregarding the not.

1		$\neg\, \forall x. \exists y. \neg\, P(x,y)$	
2	a	$\neg\, \exists y. \neg\, P(a,y))$	∀-E, 1
3			

3. Applying existential elimination disregarding both the not and the for all outside it.

1		$\neg\, \forall x. \exists y. \neg\, P(x,y)$	
2	a	$\neg\, \forall x. \neg\, P(x,a))$	∃-E, 1
3			

4. Applying all intro under an assumption.

1			$\neg\, \forall x. \exists y. \neg\, P(x,y)$	
2	a	b	$P(a,b)$	
3			$\exists y. P(a,y)$	∃-I, 3
4			$\forall x. \exists y. \neg\, P(x,y)$	∀-I, 4

5. Applying “R” (reiteration) to take expressions out of assumptions.

1			$\neg\, \forall x. \exists y. \neg\, P(x,y)$	
2	a	b	$P(a,b)$	
3			$\exists y. P(a,y)$	∃-I, 2
4			$\forall x. \exists y. \neg\, P(x,y)$	∀-I, 3
5			$\forall x. \exists y. \neg\, P(x,y)$	R, 4

Q4: Structural Induction

1. Assuming for all l $P(l)$ which is the same as the goal with a variable change. Or making the IH for an arbitrary l, $P(l)$.
2. Using a single assumption and proving only for the instance the tree was symmetric.
3. Missing the for all x in the step case $P(Node\ l\ x\ r)$.
4. Not writing the $P()$, but instead writing the induction principle or the goal.
5. Not explicitly stating what was being proved. Or defining $P(t)$ to be the entire goal.
6. Using empty list [] instead of Nul.

Q6: Hoare Logic (proof)

1. Justifying a triple with a sequence by the Assignment rule. E.g., the validity of triple $\{P\}S1;\ S2\{Q\}$ cannot be established by applying the Assignment rule as the statement is a sequence.
2. Applying the Precondition Strengthening/Postcondition Weakening/Predicate Equivalence without establishing the (bi-)implication.
3. Incorrectly justifying implications by Logic. E.g. $x > y \to x > 0 \land y > 0$ cannot be established by Logic. Counter example, $x = -10$ and $y = -100$.
4. Applying the Sequence rule incorrectly to be able to establish the premisses for the Conditional rule. E.g., having proved $\{P\}S1\{M\}$ and $\{M\}S2\{Q\}$ you can safely conclude that $\{P\}S1;\ S2\{Q\}$ is valid by the Sequence rule. But it would be wrong to state that $\{P\}S2;\ S1\{Q\}$ is valid by the Sequence rule.