



# COMP1730/COMP6730

## Programming for Scientists

### Functions (part 2)



# Lecture outline

- \* Recap of functions.
- \* Namespaces & references.
- \* Recursion revisited.



# Functions (recap)

- \* A *function* is a piece of code that can be *called* by its name.
- \* Why use functions?
  - **Abstraction**: To use a function, we only need to know *what* it does, *not how*.
  - Readability.
  - Divide and conquer – break a complex problem into simpler problems.
  - A function is a logical unit of testing.
  - Reuse: Write once, use many times (and by many).



# Function call

- \* To call a function, write its name followed by its *arguments* in parentheses:

```
change_in_percent (364, 489)
```

- \* Order of evaluation: The argument expressions are evaluated left-to-right, and their values are assigned to the parameters; then the function suite is executed.
- \* `return expression` causes the function call to end, and return the value of the expression.

# Functions without return

- \* A function call is an expression: its value is the value `return`'d by the function.
- \* In python, functions always return a value: If execution reaches the end of a function suite without executing a `return` statement, the return value is the special value `None` of type `NoneType`.
- \* **Note:** `None`-values are not printed in the interactive shell (unless explicitly with `print`).



# Namespaces

# Namespaces

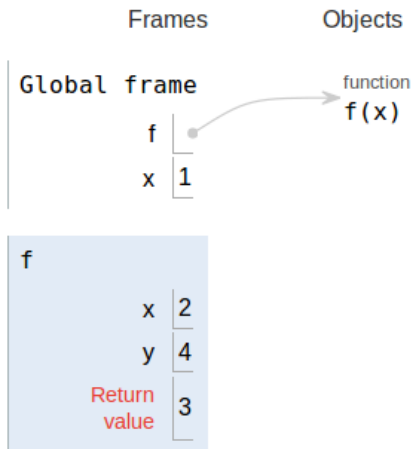
- \* Assignment associates a (variable) name with a reference to a value.
  - This association is stored in a *namespace* (sometimes also called a “*frame*”).
- \* Whenever a function is called, a new *local namespace* is created.
- \* Assignments to variables (including parameters) during execution of the function are done in the local namespace.
- \* The local namespace disappears when the function call ends.



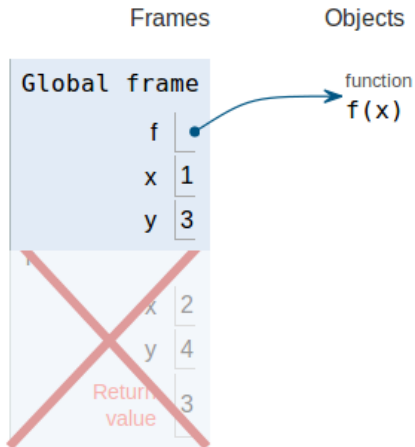
# Scope

- \* The *scope* of a variable is “the set of program statements over which a variable exists (i.e., can be referred to)”.
- In other words, the set of program statements over which the namespace that the variable is defined in persists.
- \* Because there are several namespaces, there can be *different variables with the same name in different scopes*.

```
def f(x):  
    y = x ** 2  
    return y - 1  
  
x = 1  
y = f(x + 1)
```



```
def f(x):  
    y = x ** 2  
    return y - 1  
  
x = 1  
y = f(x + 1)
```



# The local assignment rule

- \* python considers a variable that is assigned **anywhere** in the function suite to be a “*local variable*” (this includes parameters).
- \* When a non-local variable is evaluated, its value is taken from the (enclosing) global namespace.
- \* When a local variable is evaluated, only the local namespace is checked.
  - If the variable is not defined there, python raises an `UnboundLocalError`.
- \* The rule considers only *variable assignment*.

```
def f(x):  
    return x ** y  
  
>>> y = 2  
>>> f(2)  
4
```

```
def f(x):  
    if y < 1:  
        y = 1  
    return x ** y  
  
>>> y = 2  
>>> f(2)
```

UnboundLocalError:  
local variable 'y'  
referenced before  
assignment



- \* Modifying is not assignment!
  - Assignment changes/creates the association between a name and a reference (in the current namespace).
  - A modifying operation on a mutable object – including index and slice assignment – does not change any name–value association.



```
def f(x):  
    y = x ** 2  
    f_list.append([x,y])  
    return y
```

```
>>> f_list = []  
>>> f(2)  
4  
>>> f(3)  
9  
>>> f_list  
[[2, 4], [3, 9]]
```

# Argument values are references

- \* When a function is called, its parameters are assigned *references* to the argument values.
  - If an argument value refers to a mutable object (for example, a list), modifications to this object made in the function are visible outside the function's scope.





```
def f(ns):
    total = 0
    while len(ns) > 0:
        next = ns.pop(0)
        total = total + next
    return total

>>> a_list = [1,2,3]
>>> f(a_list)
6
>>> a_list
[]
```

Frames

Objects

Global frame

f  
a\_listfunction  
f(ns)

list

0	1	2
1	2	3

```
def f(ns):  
    total = 0  
    while len(ns) > 0:  
        next = ns.pop(0)  
        total = total + next  
    return total
```

```
>>> a_list = [1,2,3]  
>>> l_sum = f(a_list)
```

Image from [pythontutor.com](http://pythontutor.com)

# Other namespaces

- ★ python's built-in functions are defined in a separate namespace; it is always searched last if a name is not found elsewhere.
- ★ Imported modules are executed in their own namespace.
  - Names in a module namespace are accessed by prefixing the name of the module.
- ★ User-defined classes and objects (not covered in this course) also have their own namespace

# Guidelines for good functions

- \* Within a function, *access only local variables*.
  - Use parameters for all inputs to the function.
  - Return all function outputs (for multiple outputs, return a tuple or list).
  - ...except if the *specific purpose* of the function is to send output elsewhere (e.g., print).
- \* Don't modify mutable argument values, unless the *specific purpose* of the function is to do that.
- \* **Rule #4:** No rule should be followed off a cliff.



# Recursion

- \* A recursive function is often described as “a function that calls itself”.
- \* Function calls form a *stack*: when the  $i$ th function call ends, execution returns to where the call was made in the  $(i - 1)$ th function suite.
- \* The function suite must have a branching statement, such that a recursive call does not always take place (“base case”); otherwise, recursion never ends.
- \* Recursion is a way to think about how to solve problems: reducing it to a smaller instance of itself.

## Example (contrived)

```
def f(x):  
    '''Returns 2 ** x.  
    x is an integer >= 0.  
    '''  
    if x == 0:  
        return 1          # base case  
    else:  
        y = f(x - 1)      # recursive call  
        return 2 * y
```

```
1 def f(x):
```

```
    ...
```

```
2 y = f(2)
```

```
    x = 2
```

```
3 if x == 0:
```

```
4 else:
```

```
5 y = f(x - 1)
```

```
    x = 1
```

```
6 if x == 0:
```

```
7 else:
```

```
8 y = f(x - 1)
```

```
    x = 0
```

```
9 if x == 0:
```

```
10 return 1
```

```
    x = 1, y = 1
```

```
11 return 2 * y
```

```
    x = 2, y = 2
```

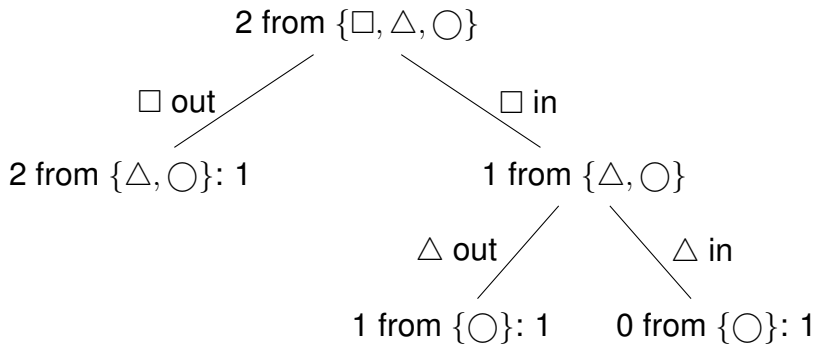
```
12 return 2 * y
```

```
y = 4
```



# Example: Counting selections

- \* Compute the number of ways to choose a subset of  $k$  elements from a set of  $n$ ,  $C(n, k)$ .





\* Recursive formulation:

$$C(n, k) = C(n - 1, k) + C(n - 1, k - 1)$$

$$C(n, 0) = 1$$

$$C(n, n) = 1$$

```
def choices(n, k):  
    if k == n or k == 0:  
        return 1  
    else:  
        return choices(n - 1, k) + \  
                choices(n - 1, k - 1)
```



```
1 ans = choices(3,2)
```

```
    n = 3, k = 2
```

```
2 if k == 0 or k == n:
```

```
3 else:
```

```
4 choices(n - 1, k)
```

```
    n = 2, k = 2
```

```
5 if k == 0 or k == n:
```

```
6 return 1
```

```
7 choices(n - 1, k - 1)
```

```
    n = 2, k = 1
```

```
8 if k == 0 or k == n:
```

```
9 else:
```

```
10 choices(n - 1, k)
```

```
    n = 1, k = 1
```

```
11 if k == 0 or k == n:
```

```
12 return 1
```

```
13 choices(n - 1, k - 1)
```

```
    n = 1, k = 0
```

```
14 if k == 0 or k == n:
```

```
15 return 1
```



```
4 choices(n - 1, k)
```

```
    n = 2, k = 2
```

```
    | 5 if k == 0 or k == n:
```

```
    | 6 return 1
```

```
7 choices(n - 1, k - 1)
```

```
    n = 2, k = 1
```

```
    | 8 if k == 0 or k == n:
```

```
    | 9 else:
```

```
    | 10 choices(n - 1, k)
```

```
        n = 1, k = 1
```

```
        | 11 if k == 0 or k == n:
```

```
        | 12 return 1
```

```
    | 13 choices(n - 1, k - 1)
```

```
        n = 1, k = 0
```

```
        | 14 if k == 0 or k == n:
```

```
        | 15 return 1
```

```
    | 16 return 1 + 1
```

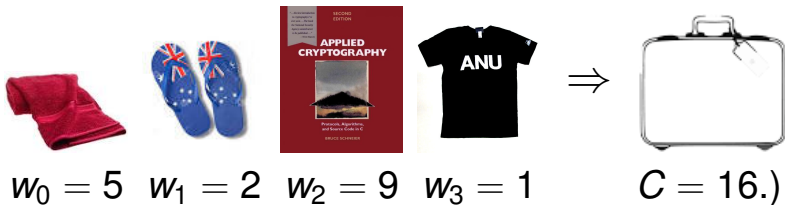
```
17 return 1 + 2
```

```
ans = 3
```

# Example: Subset sum

- ★ Given a list of  $n$  integers  $w_0, \dots, w_{n-1}$ , is there a subset of them that sums to exactly  $C$ ?

(Also known as the “(exact) knapsack problem”:



# Example: Sudoku

