ENGN2219/COMP6719
Computer Systems & Organization
Problem Set 2

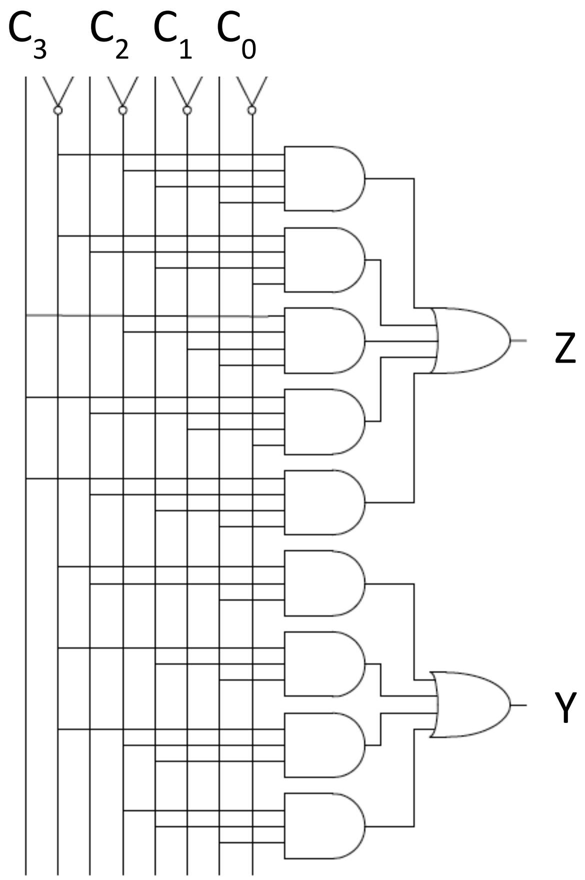
Note: This problem set is optional for your practice only and not part of the assessment scheme.

Question 1:

Prove De Morgan's Theorem (T12) for three variables, A_2, A_1, A_0 using perfect induction.

Question 2:

A circuit has four inputs and two outputs. The inputs $C_{3:0}$ represent a number from 0 to 15. Output Y should be TRUE if the number is prime (0 and 1 are not prime, but 2, 3, 5, and so on, are prime). Output Z should be TRUE if the number is divisible by 3. Give simplified Boolean equations for each output and sketch a circuit.

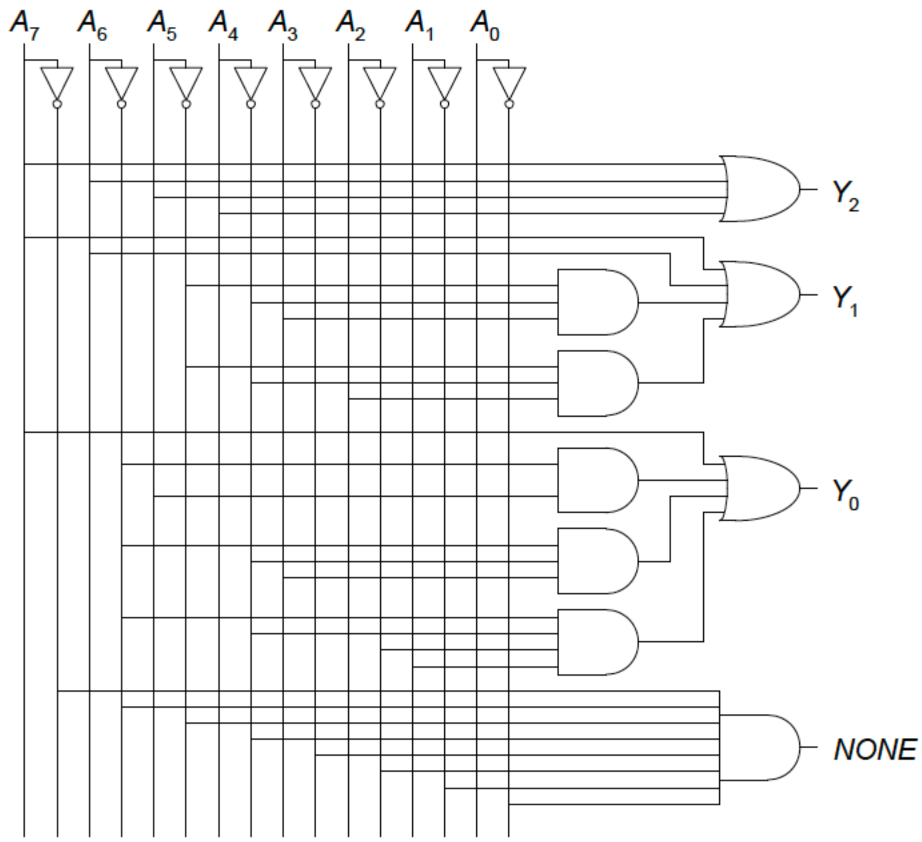


See next page for equations and truth table.

Decimal Value	C_3	C_2	C_1	C_0	Z	Y
0	0	0	0	0	0	0
1	0	0	0	1	0	0
2	0	0	1	0	0	1
3	0	0	1	1	1	1
4	0	1	0	0	0	0
5	0	1	0	1	0	1
6	0	1	1	0	1	0
7	0	1	1	1	0	1
8	1	0	0	0	0	0
9	1	0	0	1	1	0
10	1	0	1	0	0	0
11	1	0	1	1	0	1
12	1	1	0	0	1	0
13	1	1	0	1	0	1
14	1	1	1	0	0	0
15	1	1	1	1	1	0

Question 3:

A priority encoder has 2^N inputs. It produces an N-bit binary output indicating the most significant bit of the input that is TRUE, or 0 if none of the inputs are TRUE. It also produces an output NONE that is TRUE if none of the inputs are TRUE. Design an eight-input priority encoder with inputs $A_{7:0}$ and outputs $Y_{2:0}$ and NONE. For example, if the input is 00100000, the output Y should be 101 and NONE should be 0. Give a simplified Boolean equation for each output, and sketch a schematic.



See next page for equations and truth table.

A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0	Y_2	Y_1	Y_0	$NONE$
0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	1	X	0	0	1	0
0	0	0	0	0	1	X	X	0	1	0	0
0	0	0	0	1	X	X	X	0	1	1	0
0	0	0	1	X	X	X	X	1	0	0	0
0	0	1	X	X	X	X	X	1	0	1	0
0	1	X	X	X	X	X	X	1	1	0	0
1	X	X	X	X	X	X	X	1	1	1	0

$$Y_2 = A_7 + A_6 + A_5 + A_4$$

$$Y_1 = A_7 + A_6 + \overline{A_5}\,\overline{A_4}A_3 + \overline{A_5}\,\overline{A_4}A_2$$

$$Y_0 = A_7 + \overline{A_6}A_5 + \overline{A_6}\,\overline{A_4}A_3 + \overline{A_6}\,\overline{A_4}\,\overline{A_2}A_1$$

$$NONE = \overline{A_7}\,\overline{A_6}\,\overline{A_5}\,\overline{A_4}\,\overline{A_3}\,\overline{A_2}\,\overline{A_1}\,\overline{A_0}$$

Question 4:

Write a Boolean equation in sum-of-products canonical form for the following truth table. Minimize the equation.

Y = A’B’C’ + A’BC’ + AB’C’ + AB’C + ABC

Y = A’C’ + B’C’ + AC

Can you do a better minimization?

Question 5:

Write a Boolean equation in sum-of-products canonical form for the following truth table. Minimize the equation.

$$Y = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}B\overline{C}D + \overline{A}BC\overline{D} + A\overline{B}\,\overline{C}D + A\overline{B}C\overline{D} + AB\overline{C}\,\overline{D} + ABCD$$

Minimization is up to you

Question 6:

Write Boolean equations for the circuit below. Do not minimize the equations.

$$Y = \overline{A}D + A\overline{B}C + A\overline{C}D + ABCD$$
$$Z = A\overline{C}D + BD$$

Question 7:

Use De Morgan equivalent gates and bubble pushing rules to redraw the circuit below. The goal is to simplify the circuit so that the Boolean equation is obvious by inspection. Write the Boolean equation.

$$Y = (\overline{A} + \overline{B})(\overline{C} + \overline{D}) + \overline{E}$$

Question 8:

Write a minimized Boolean equation for the function performed by the circuit below. (Note that the upside-down triangle sign to which multiplexer inputs 01 and 10 are connected represent a permanent connection to 0. And the dash sign to which 00 and 11 are connected represents a permanent connection to 1.)

$$Y = A + \overline{C \oplus D} = A + CD + \overline{C}\overline{D}$$