Exercise 3.26



Note:  $\overline{N} \bullet \overline{D} \bullet \overline{Q} = \overline{Nickel} \bullet \overline{Dime} \bullet \overline{Quarter}$ 

FIGURE 3.2 State transition diagram for soda machine dispense of Exercise 3.23

state	e n c o d i n g <sup>S</sup> 9 : 0
S0	000000001
S5	000000010
S10	000000100
S25	0000001000
<b>S</b> 30	0000010000
S15	0000100000
S20	0001000000
S35	0010000000
S40	010000000
S45	100000000

FIGURE 3.3 State Encodings for Exercise 3.26

current		inputs		
state	nickel	d i m e	quarter	state s'
<b>S</b> 0	0	0	0	<b>S</b> 0
<b>S</b> 0	0	0	1	S25
<b>S</b> 0	0	1	0	S10
S0	1	0	0	S5
S5	0	0	0	S5
S5	0	0	1	S30
S5	0	1	0	S15
S5	1	0	0	S10
S10	0	0	0	S10

TABLE 3.11 State transition table for Exercise 3.26

current		inputs		n e x t
state	nickel	d i m e	quarter	state s'
S10	0	0	1	S35
S10	0	1	0	S20
S10	1	0	0	S15
S25	Х	Х	Х	<b>S</b> 0
<b>S</b> 30	Х	Х	Х	<b>S</b> 0
S15	0	0	0	S15
S15	0	0	1	<b>S</b> 40
S15	0	1	0	S25
S15	1	0	0	S20
S20	0	0	0	S20
S20	0	0	1	S45
S20	0	1	0	<b>S</b> 30
S20	1	0	0	S25
S35	X	Х	Х	SO
S40	X	Х	Х	SO
S45	Х	Х	Х	SO

 TABLE 3.11
 State transition table for Exercise 3.26

current		inputs next state		
state	nickel	d i m e	quarter	S
000000001	0	0	0	000000001
000000001	0	0	1	0000001000
000000001	0	1	0	000000100
000000001	1	0	0	000000010

 TABLE 3.12
 State transition table for Exercise 3.26

current	inputs		nextstate	
state	nickel	d i m e	quarter	S '
000000010	0	0	0	000000010
000000010	0	0	1	0000010000
000000010	0	1	0	0000100000
000000010	1	0	0	000000100
000000100	0	0	0	000000100
000000100	0	0	1	0010000000
000000100	0	1	0	0001000000
000000100	1	0	0	0000100000
0000001000	Х	Х	Х	000000001
0000010000	Х	Х	Х	0000000001
0000100000	0	0	0	0000100000
0000100000	0	0	1	010000000
0000100000	0	1	0	0000001000
0000100000	1	0	0	0001000000
0001000000	0	0	0	0001000000
0001000000	0	0	1	100000000
0001000000	0	1	0	0000010000
0001000000	1	0	0	0000001000
0010000000	X	X	Х	0000000001
010000000	Х	Х	Х	0000000001
100000000	Х	Х	Х	000000001

<b>TABLE 3.12</b>	State transition	table fo	r Exercise 3.26
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$$S_9 = S_6 Q$$
$$S_8 = S_5 Q$$

$$S_{7} = S_{2}Q$$

$$S_{6} = S_{2}D + S_{5}N + S_{6}\overline{NDQ}$$

$$S_{5} = S_{1}D + S_{2}N + S_{5}NDQ$$

$$S_{4} = S_{1}Q + S_{6}D$$

$$S_{3} = S_{0}Q + S_{5}D + S_{6}N$$

$$S_{2} = S_{0}D + S_{1}N + S_{2}\overline{NDQ}$$

$$S_{1} = S_{0}N + S_{1}NDQ$$

$$S_{0} = S_{0}\overline{NDQ} + S_{3} + S_{4} + S_{7} + S_{8} + S_{9}$$
Dispense =  $S_{2} + S_{4} + S_{7} + S_{8} + S_{9}$ 

 $Dispense = S_3 + S_4 + S_7 + S_8 + S_9$ ReturnNickel =  $S_4 + S_8$ ReturnDime =  $S_7 + S_8$ ReturnTwoDimes =  $S_9$  SOLUTIONS chapter 3



Exercise 3.27



FIGURE 3.4 State transition diagram for Exercise 3.27

current state <sup>s</sup> 2:0	n e x t s t a t e s'2:0
000	001
001	011
011	010
010	110
110	111
111	101
101	100
100	000

 TABLE 3.13
 State transition table for Exercise 3.27

$$S'_{2} = S_{1}\overline{S_{0}} + S_{2}S_{0}$$
$$S'_{1} = \overline{S_{2}}S_{0} + S_{1}\overline{S_{0}}$$
$$S'_{0} = \overline{S_{2} \oplus S_{1}}$$
$$Q_{2} = S_{2}$$
$$Q_{1} = S_{1}$$
$$Q_{0} = S_{0}$$



FIGURE 3.5 Hardware for Gray code counter FSM for Exercise 3.27

Exercise 3.28



FIGURE 3.6 State transition diagram for Exercise 3.28

current state <sup>s</sup> 2 : 0	input up	next state s'2:0
000	1	001
001	1	011
011	1	010
010	1	110
110	1	111
111	1	101
101	1	100
100	1	000
000	0	100
001	0	000
011	0	001
010	0	011
110	0	010
111	0	110
101	0	111
100	0	101

<b>TABLE 3.14</b>	State trans	sition table	for	Exercise	3.28

$$S_{2} = UPS_{1}\overline{S_{0}} + \overline{UP}\overline{S_{1}}\overline{S_{0}} + S_{2}S_{0}$$

$$S_{1} = S_{1}\overline{S_{0}} + UP\overline{S_{2}}S_{0} + \overline{UP}S_{2}S_{1}$$

$$S_{0} = UP \oplus S_{2} \oplus S_{1}$$

$$Q_{2} = S_{2}$$

$$Q_{1} = S_{1}$$

$$Q_{0} = S_{0}$$



FIGURE 3.7 Finite state machine hardware for Exercise 3.28



FIGURE 3.8 Waveform showing Z output for Exercise 3.29

(b) This FSM is a Mealy FSM because the output depends on the current value of the input as well as the current state.

(c)





(Note: another viable solution would be to allow the state to transition from S0 to S1 on  $B\overline{A}/0$ . The arrow from S0 to S0 would then be  $\overline{B}\overline{A}/0$ .)

current state	inputs next state ou		output	
<sup>3</sup> 1:0	b	а	$S^{-1} : 0$	Z
00	Х	0	00	0
00	0	1	11	0
00	1	1	01	1
01	0	0	00	0
01	0	1	11	1
01	1	0	10	1
01	1	1	01	1
10	0	Х	00	0
10	1	0	10	0

 TABLE 3.15
 State transition table for Exercise 3.29

current state	i n p u	i t s	nextstate	output
<sup>5</sup> 1 : 0	b	а	<i>S</i> <sup>'</sup> 1 : 0	Z
10	1	1	01	1
11	0	0	00	0
11	0	1	11	1
11	1	0	10	1
11	1	1	01	1

TABLE 3.15 State transition table for Exercise 3.29
---

$$S_1 = \overline{B}A(\overline{S_1} + S_0) + B\overline{A}(S_1 + \overline{S_0})$$
$$S_0 = A(\overline{S_1} + S_0 + B)$$

$$Z = BA + S_0(A + B)$$



FIGURE 3.10 Hardware for FSM of Exercise 3.26

**Note:** One could also build this functionality by registering input *A*, producing both the logical AND and OR of input *A* and its previous (registered)

value, and then muxing the two operations using *B*. The output of the mux is *Z*: Z = AA prev (if B = 0); Z = A + A prev (if B = 1).

## Exercise 3.30



FIGURE 3.11 Factored state transition diagram for Exercise 3.30

current state <sup>s</sup> 1:0	input a	next state s'1:0
00	0	00
00	1	01
01	0	00

TABLE 3.16 State transition table for output *Y* for Exercise 3.30

current state <sup>s</sup> 1:0	input a	next state s'1:0
01	1	11
11	Х	11

 TABLE 3.16
 State transition table for output Y for Exercise 3.30

current state t <sub>1:0</sub>	input a	next state t' <sub>1:0</sub>
00	0	00
00	1	01
01	0	01
01	1	10
10	0	10
10	1	11
11	Х	11

 TABLE 3.17
 State transition table for output X for Exercise 3.30

$$S_{1} = S_{0}(S_{1} + A)$$

$$S_{0} = \overline{S_{1}}A + S_{0}(S_{1} + A)$$

$$Y = S_{1}$$

$$T_{1} = T_{1} + T_{0}A$$

$$T_0 = A(T_1 + \overline{T_0}) + \overline{A}T_0 + T_1T_0$$
$$X = T_1T_0$$



FIGURE 3.12 Finite state machine hardware for Exercise 3.30

## Exercise 3.31

This finite state machine is a divide-by-two counter (see Section 3.4.2) when X = 0. When X = 1, the output, Q, is HIGH.

current state		input	next state	
<i>s</i> <sub>1</sub>	<i>s</i> 0	x	<i>s</i> ' <sub>1</sub>	<i>s</i> ' <sub>0</sub>
0	0	0	0	1
0	0	1	1	1
0	1	0	0	0

TABLE 3.18 State transition table with binary encodings for Exercise 3.31

current state		input	next state	
<i>s</i> <sub>1</sub>	<i>s</i> <sub>0</sub>	x	<i>s</i> ' <sub>1</sub>	<i>s</i> ' <sub>0</sub>
0	1	1	1	0
1	Х	Х	0	1

TABLE 3.18 State transition table with binary encodings for Exercise 3.31

current state		output	
s <sub>1</sub>	s <sub>0</sub>	q	
0	0	0	
0	1	1	
1	Х	1	

TABLE 3.19 Output table for Exercise 3.31



Exercise 3.32

current state		input	next state			
<i>s</i> <sub>2</sub>	<i>s</i> <sub>1</sub>	<i>s</i> 0	а	<i>s</i> ' <sub>2</sub>	<i>s</i> ' <sub>1</sub>	<i>s</i> ' <sub>0</sub>
0	0	1	0	0	0	1
0	0	1	1	0	1	0
0	1	0	0	0	0	1

TABLE 3.20 State transition table with binary encodings for Exercise 3.32

current state		input	next state			
<i>s</i> <sub>2</sub>	<i>s</i> <sub>1</sub>	<i>s</i> 0	а	<i>s</i> ' <sub>2</sub>	<i>s</i> ' <sub>1</sub>	<i>s</i> ' <sub>0</sub>
0	1	0	1	1	0	0
1	0	0	0	0	0	1
1	0	0	1	1	0	0

TABLE 3.20 State transition table with binary encodings for Exercise 3.32



FIGURE 3.13 State transition diagram for Exercise 3.32

Q asserts whenever A is HIGH for two or more consecutive cycles.

## Exercise 3.33

ic:

(a) First, we calculate the propagation delay through the combinational log-

 $t_{pd} = 3t_{pd} \operatorname{XOR}$ = 3 × 100 ps = **300 ps** Next, we calculate the cycle time:  $T_c \ge t_{pcq} + t_{pd} + t_{setup}$  $\ge [70 + 300 + 60] \text{ ps}$ = 430 ps f = 1 / 430 ps = 2.33 GHz(b)  $T_c \ge t_{pcq} + t_{pd} + t_{setup} + t_{skew}$ Thus,  $t_{skew} \le T_c - (t_{pcq} + t_{pd} + t_{setup}), \text{ where } T_c = 1 / 2 \text{ GHz} = 500 \text{ ps}$  $\le [500 - 430] \text{ ps} = 70 \text{ ps}$ 



(b) Each tag is 16 bits. There are 32Kwords / (2 words / block) = 16K blocks and each block needs a tag: 16 × 16K = 218 = **256 Kbits** of tags.

(c) Each cache block requires: 2 status bits, 16 bits of tag, and 64 data bits, thus each set is  $2 \times 82$  bits = **164 bits**.

(d) See figure below. The design must use enough RAM chips to handle both the total capacity and the number of bits that must be read on each cycle. For the data, the SRAM must provide a capacity of 128 KB and must read 64 bits per cycle (one 32-bit word from each way). Thus the design needs at least 128KB / (8KB/RAM) = 16 RAMs to hold the data and 64 bits / (4 pins/RAM) = 16 RAMs to supply the number of bits. These are equal, so the design needs exactly 16 RAMs for the data.

For the tags, the total capacity is 32 KB, from which 32 bits (two 16-bit tags) must be read each cycle. Therefore, only 4 RAMs are necessary to meet the capacity, but 8 RAMs are needed to supply 32 bits per cycle. Therefore, the design will need 8 RAMs, each of which is being used at half capacity.

With 8K sets, the status bits require another  $8K \times 4$ -bit RAM. We use a  $16K \times 4$ -bit RAM, using only half of the entries.