1.1

p	q	$p \wedge q \to \neg p$		
1	1	1	0	0
1	0	0	1	0
0	1	0	1	1
0	0	0	1	1

Yes: there are rows in which it is 1.

1.2

p	q	r	$\neg p \vee q$	$\neg(q \wedge \neg r)$		$p \to r$	
1	1	1	1	1	0	1	←
1	1	0	1	0	1	0	
1	0	1	0	1	0	1	
1	0	0	0	1	0	0	
0	1	1	1	1	0	1	←
0	1	0	1	0	1	1	
0	0	1	1	1	0	1	←
0	0	0	1	1	0	1	←

Yes: The rows on which both premises are 1 also have conclusion 1.

2.1

α_1	(1)	$\neg\neg\neg p$	A
α_2	(2)	p	A
α_3	(3)	$\neg p$	A
α_2, α_3	(4)	$\bot$	2, 3, $\neg$E
α_2	(5)	$\neg\neg p$	4, $\neg$I
α_1, α_2	(6)	$\bot$	1, 5, $\neg$E
α_1	(7)	$\neg p$	6, $\neg$I
	(8)	$\neg\neg\neg p \to \neg p$	7, $\neg$I

2.2

α_1	(1)	$(p \to q) \vee \neg\neg r$	A
α_2	(2)	p	A
α_3	(3)	$p \to q$	A
α_2, α_3	(4)	q	2, 3, $\to$E
α_2, α_3	(5)	$q \vee r$	4, $\vee$I1
α_6	(6)	$\neg\neg r$	A
α_6	(7)	r	6, $\neg\neg$E
α_6	(8)	$q \vee r$	7, $\vee$I2
α_1, α_2	(9)	$q \vee r$	1, 5, 8, $\vee$E

2.3

α_1	(1)	$\forall x \forall y (Rxy \to x = y)$	A
α_2	(2)	$\exists x \neg (x = c)$	A
α_3	(3)	$\neg (a = c)$	A
α_1	(4)	$\forall y (Ray \to a = y)$	1, $\forall$E
α_1	(5)	$Rac \to a = c$	4, $\forall$E
α_6	(6)	Rac	A
α_1, α_6	(7)	$a = c$	5, 6, $\to$E
$\alpha_1, \alpha_3, \alpha_6$	(8)	$\bot$	3, 7, $\neg$E
α_1, α_3	(9)	$\neg Rac$	8, $\neg$I
α_1, α_3	(10)	$\exists y \neg Ray$	9, $\exists$I
α_1, α_3	(11)	$\exists x \exists y \neg Rxy$	10, $\exists$I
α_1, α_2	(12)	$\exists x \exists y \neg Rxy$	2, 11, $\exists$E

3.1 Assume that
- For any truth table row evaluating all of Γ to 1, $\varphi \to \psi$ is 1
- For any truth table row evaluating all of Γ' and ψ to 1, θ is 1.

Consider a row that evaluates all of Γ, Γ', and φ to 1.
By the first assumption, $\varphi \to \psi$ is 1.
By the truth table for $\to$, if φ is 1 and $\varphi \to \psi$ is 1 then ψ is 1
Then by the second assumption, θ is 1.

3.2 Assume that
- the (a) and (b) statements hold both for some φ, and also for some other proposition ψ.

(a) Suppose that $\varphi \wedge \psi$ has truth table evaluation 1. Then by assumption $\vdash \varphi$ and $\vdash \psi$ have proofs (because by the truth table, both φ and ψ are 1). We can combine these proofs with $\wedge$I to get $\vdash \varphi \wedge \psi$.

(b) Suppose that $\varphi \wedge \psi$ has truth table evaluation 0. Then by the truth table at least one of φ and ψ is 0. Without loss of generality, say φ is 0. Then we have

	(1)	$\neg \varphi$	by induction hypothesis
α_2	(2)	$\varphi \wedge \psi$	A
α_2	(3)	φ	2, $\wedge$E
α_2	(4)	$\bot$	1, 3, $\neg$E
	(5)	$\neg(\varphi \wedge \psi)$	4, $\neg$I

3.3 Assume $\models_{M,e} \varphi(t/y)$ iff $\models_{M,e(y\mapsto t^{M,e})} \varphi$

We will leave M out to avoid clutter:

$\models_{e} (\exists^{\infty}_{x} \varphi)(t/y)$

iff $\models_e \exists^{\infty}_{x} (\varphi(t/y))$ - assume $x \neq y$, x not free in t

iff for infinitely many d, $\models_{e(x\mapsto d)} \varphi(t/y)$

iff for infinitely many d, $\models_{e(x\mapsto d)(y\mapsto t^{e(x\mapsto d)})} \varphi$ (Induction)...

But by the side conditions to substitution, $e(x\mapsto d)(y\mapsto t^{e(x\mapsto d)})$ is $e(y\mapsto t^{e})(x\mapsto d)$, so our statement holds.

iff $\models_{e(y\mapsto t^e)(x\mapsto d)} \varphi$ for infinitely many d

iff $\models_{e(y\mapsto t^e)} \exists^{\infty}_{x} \varphi$

4.1 m = matter is a necessary being

g^+ = gravitation is necessarily present

g^- = gravitation is necessarily absent

i = the world is subject to a presiding intelligence

o = motion exists

v = a vacuum is necessary

- $m \to g^+ \vee g^-$
- $g^- \wedge \neg i \to \neg o$
- $g^+ \to v$
- $v \to \neg m$
- $m \to \neg i$

The missing premise is $\cdot\, o$

(No explanation is asked for, but: $m \to g^+ \vee g^-$. But $g^+ \to v$ and $v \to \neg m$, so $g^+ \to \neg m$. So we can reject $m \to g^+$. and conclude $m \to g^-$. Further $m \to \neg i$, so $m \to g^- \wedge \neg i$. But the contrapositive of the second premise is $o \to \neg(g^- \wedge \neg i)$. So with the missing premise, $\neg(g^- \wedge \neg i)$, so $\neg m$.)

4.2 I will answer in Logic4Fun syntax

Sorts : c. (for creature)

h. (for house)

Vocab : function r (creature) : house. (resides)

predicate g (creature). (ghost)

predicate s (creature, creature). (scares)

Constraints : • ALL x (NOT SOME y (x <> y AND r x = r y) IMP NOT SOME y (x s y)).

• ALL x (x s x IMP NOT SOME y (g y AND r x = r y))

• ALL x (g x AND ALL y (x s y IMP NOT g y) IMP
ALL y (r x = r y AND NOT g y IMP x s y)).

• ALL x (ALL y (x s y IMP r x <> r y) IMP
ALL y (x s y IMP g y) OR ALL y (x s y IMP NOT g y)).

• SOME x SOME y (x <> y AND g x AND g y AND x s y AND y s x)
AND ALL x ALL y (x <> y AND g x AND g y AND x s y AND y s x
IMP r x = r y).

4.3 • $G(a \vee d \vee h \vee p)$

• $Fd \wedge \neg d$

• $Fp \rightarrow FGp$

• $G(a \wedge d \rightarrow X(a \wedge \neg d))$

• $\neg h \; U \; (G\neg a \wedge h)$

• $AG(e \rightarrow EX\neg e)$

• $AG(a \wedge p \rightarrow e) \wedge AGEF(a \wedge p)$

5.1 (1) T : $(p \rightarrow q) \rightarrow p$ ✓

(2) T : $q \wedge r \rightarrow \neg p$ ✓

(3) F : $\neg q \vee \neg r$ ✓

(4) F : $\neg q$ from (3) ✓

(5) F : $\neg r$ from (3) ✓

(6) T : q from (4)

(7) T : r from (5)

(8) F : $q \wedge r$ from (2) | (9) T : $\neg p$ from (2) ✓

(10) F : q from (8) | (11) F : r from (8) | (12) F : p from (9)

X (6, 10) | X (7, 11) | (13) F : $p \rightarrow q$ from (1) ✓ | (14) T : p from (1)

(15) T : p from (13) | X (12, 14)

X (12, 15)

5.2

(1) F : $\exists x(Dx \to \forall y Dy)$ ✓

(2) F : $Da \to \forall y Dy$ from (1) ✓

(3) T : Da from (2)

(4) F : $\forall y Dy$ from (2) ✓

(5) F : Da from (4)

X (3, 5)

5.3

(1) T : $\exists x \exists y Ryx$ ✓

(2) T : $\forall x \forall y (Rxy \to \neg Ryx)$ ✓

(3) T : $\exists y Rya$ from (1) ✓

(4) T : Rba from (3)

(5) T : $\forall y (Ray \to \neg Rya)$ from (2) ✓

(6) T : $\forall y (Rby \to \neg Ryb)$ from (2) ✓

(7) T : $Raa \to \neg Raa$ from (5) ✓

(8) T : $Rab \to \neg Rba$ from (5) ✓

(9) T : $Rba \to \neg Rab$ from (6) ✓

(10) T : $Rbb \to \neg Rbb$ from (6) ✓

(11) F : Raa from (7)

(12) F : Rbb from (10)

(13) F : Rab from (8)

(14) T : $\neg Rab$ from (9) ✓

(15) F : Rab from (14)

$D = \{a, b\}$, R is interpreted as $\{(b, a)\}$

5.4

(1) F : $\exists x (\exists y Rxy \to Rxx)$ ✓✓

(2) F : $\exists y Ray \to Raa$ from (1) ✓

(3) T : $\exists y Ray$ from (2) ✓

(4) F : Raa from (2)

(5) T : Rab from (3)

(6) F : $\exists y Rby \to Rbb$ from (1) ✓

(7) T : $\exists y Rby$ from (6) ✓

(8) F : Rbb from (6)

(9) T : Rba from (7)

$D = \{a, b\}$, R is interpreted as $\neq$.

5.5

(1) T: F(p U q) ✓

(2) F: F q ✓

(3) F: q from (2)

(4) F: X F q from (2) ✓

(5) T: p U q from (1) ✓

(7) T: p from (5)

(8) T: X(p U q) from (5) ✓

⇟

(10) F: F q from (4) ✓

(11) T: p U q from (8) ✓

(12) F: q from (10)

(13) F: X F q from (10)

(14) T: p from (11)

(15) T: X(p U q) from (11)

X (REP, 1)

(16) T: q from (11)

X (12, 16)

(9) T: q from (5)

X (3, 9)

(6) T: X F(p U q) from (1) ✓

⇟

(17) F: F q from (4)

(18) T: F(p U q) from (6)

Identical to node above

X (REP, 1)

6.1 • True: take x to be 0. Then $\neg(x = c)$ is false, so the implication is vacuously true, so the existential is true.

• False: if x is ϕ then there is no non-empty set y such that $\phi \cup y \subseteq \phi$ (because $\phi \cup y = y$).

6.3 • EG p implies AF p:

Suppose $\models_{M,s}$ EG p

Then there exists a path $s \longrightarrow \ldots$ with p labelling every state on the path.

In particular, p labels s.

But all paths from s include s, so the exists a state on all paths $s \longrightarrow \ldots$ with p, so $\models_{M,s}$ AF p

• AF p does not imply EG p:

All paths (there is only one!) eventually reach p, but no path is always p.

6.2 Does satisfy: s_0, then s_1, then s_2, then s_3, then $s_2 s_3 s_2 s_3$ forever (or, s_0 forever)

Does not satisfy: $s_0 s_1 s_3 s_2$, then $s_3 s_2$ forever

6.4, $\models_{M,s} E(XXp)$ if there exists σ with $\sigma_0 = s$ and $p \in L(\sigma_2)$.

$\models_{M,s} EXEXp$ if there exists σ' with $\sigma'_0 = s$ and there exists σ'' with $\sigma''_0 = \sigma'_1$ such that $p \in L(\sigma''_1)$

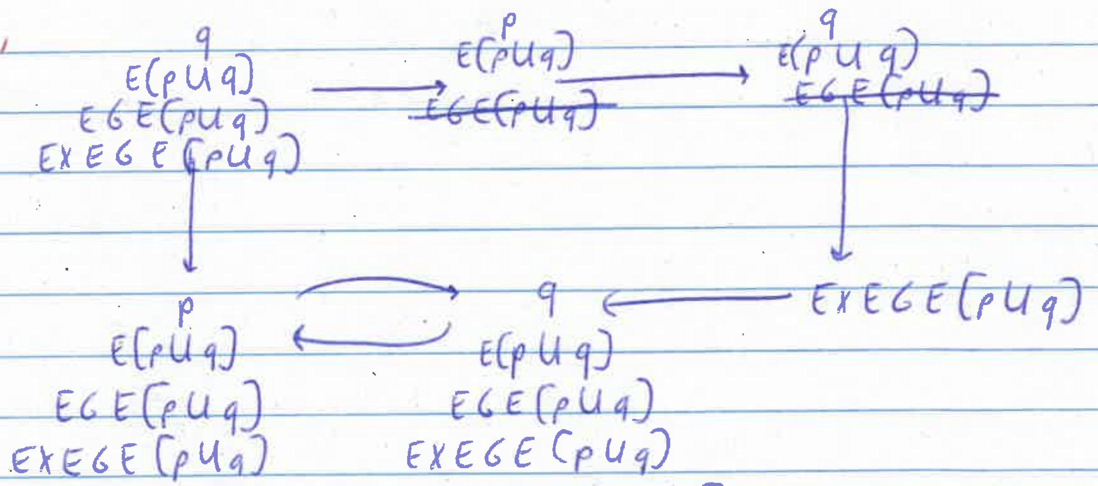
(⇒) Let σ' be σ and σ'' be $\sigma_{\geq 1}$

Then $\sigma'_0 = \sigma_0 = s$, and $\sigma''_0 = (\sigma_{\geq 1})_0 = \sigma_1 = \sigma'_1$, and $\sigma''_1 = (\sigma_{\geq 1})_1 = \sigma_2$, which contains p.

(⇐) Let σ be s followed by σ''. This is possible because there exists the path σ' whose second element is σ''_0.

Now $\sigma_2 = (\sigma'')_1$, which contains p.

7.1,



7.2, SCC 1: s_0, s_3, s_4, s_6 Fair

SCC 2: s_1 Trivial.

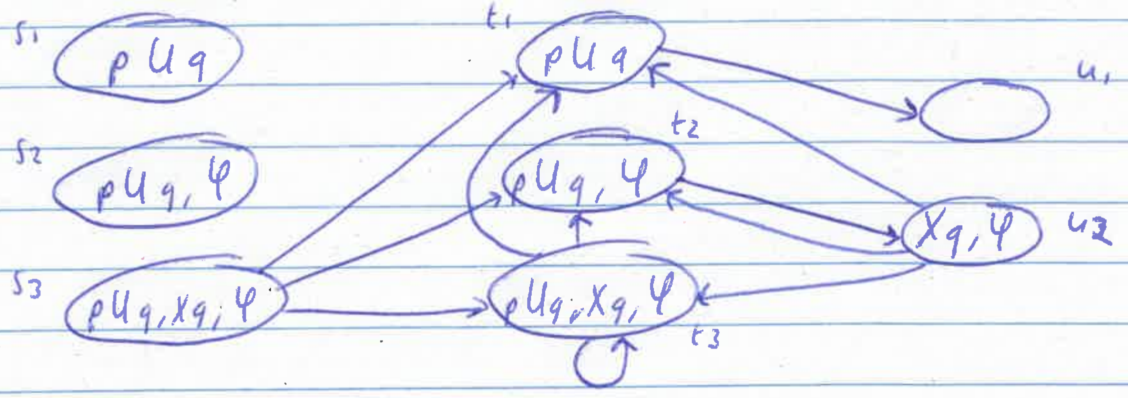
SCC 3: s_2 Fair

SCC 4: s_7

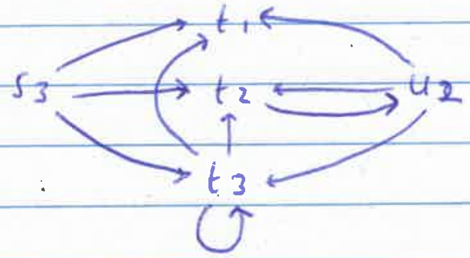
SCC 5: s_5, s_8

$E_C GT$ holds at all members of fair SCCs (s_0, s_3, s_4, s_6, s_2) and all states that can reach fair SCCs (s_1).

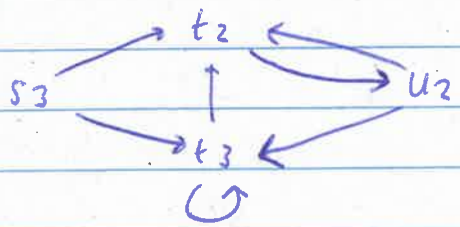
7.3 Omitting the p and q labels, and writing the main proposition as φ:



Deleting states with no out-transitions:



and again:

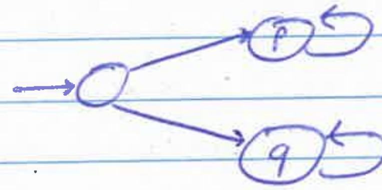


Comp 6262 Alternate Question

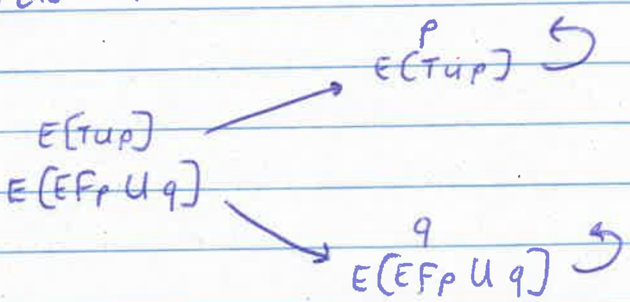
- $\models_{M,s} E[F_p U q]$ means that there exists a path σ with $\sigma_0 = s$ satisfying $F_p U q$: there exists a number i such that $q \in L(\sigma_i)$ ~~q~~, and for all natural numbers (if any) j less than i, there exists a $k \geq j$ such that $p \notin L(\sigma_k)$.

  $\models_{M,s} E[EF_p U q]$ means that there exists a path σ' with $\sigma'_0 = s$ satisfying $EF_p U q$: the first condition is the same as above, and the second states that for all natural numbers j less than i, there exists a path σ^j from σ_j, and a ~~and~~ number k such that $p \notin L(\sigma^j)_k$.

($\Rightarrow$) Let σ' be σ. Then $\sigma'_0 = s$ and there is indeed an i such that $q \in L(\sigma'_i) = L(\sigma_i)$. For each j, let σ^j be $\sigma_{\geq j}$. Then there is indeed an index for each such path with no p label.



Model check: Recall EF_p is $E[\top U p]$, where every state gets $\top$.



$E[F_p U q]$ fails because the only path that reaches q has no p in it, so F_p does not hold before we reach q.