Introduction to COMP2620/6262: Logic

Intro to Logic, Course Overview, Course Representatives

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Acknowledgement of Country

We acknowledge and celebrate the First Australians on whose traditional lands we meet, the Ngunnawal and Ngambri people, and pay our respect to the elders past and present.

Teaching Team

My name is Ranald Clouston, and I am the convenor of this course

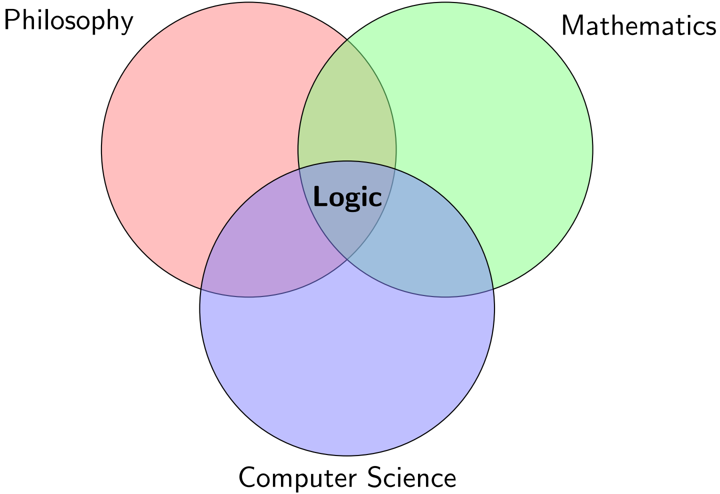
- Research: logic, type theory, category theory
- Teaching: this course, Logic Summer School, introductory programming

I will teach with an all star team of tutors!

Oscar Czernuszyn	Mary Kwan	Claire Li
Stanley Li	Xin Lu	Sophie Press
Anneysha Sarkar	Jack Stodart	
Liam van der Wyver	Mohammad Yousefi	

An Informal Introduction to Formal Logic

The Study of Logic



Logic and Philosophy

Logic began with ancient Greek, Indian, and Chinese philosophers looking for the rules governing valid arguments:

> If you accept the premises, and are rational, you must accept the conclusion

All Greeks are humans
All humans are mortal
∴ All Greeks are mortal ✓

All Greeks are humans
All humans are mortal
∴ All mortals are Greek ✗

Aristotle. c/o Wikimedia

Logic and Philosophy

We abstract away the content of the argument to focus only on its form.

All Greeks are humans
All humans are mortal
∴ All Greeks are mortal ✓

All Greeks are squirrels
All squirrels are bioluminescent
∴ All Greeks are bioluminescent ✓

All Greeks are numerals
All numerals are mortal
∴ All Greeks are mortal ✓

All monoids are semigroups
All semigroups are magmas
∴ All monoids are magmas ✓

Logic and Philosophy

We abstract by replacing our meaningful propositions and predicates with arbitrary variables:

All A are B
All B are C
$\therefore$ All A are C ✓

We usually use formal notation for the logical connections in our argument:

$\forall x. A(x) \to B(x)$
$\forall x. B(x) \to C(x)$
$\therefore\ \forall x. A(x) \to C(x)$ ✓

But it is the abstraction, not the notation, that is important.

Logic and Philosophy

Philosophers continue to contribute to the development of logic, and to use logic to organise their arguments.

If v inheres in other entities, then v is a particular and it inheres in exactly one tuple of entities (A6, A7). If v inheres in $v_1, \ldots, v_d$ and v exists at t, then $v_1, \ldots, v_d$ are substance particulars which exist at t (A8). There is no substance without qualities. Hence if a substance particular v exists at t, then there exists at t a 1-place quality which inheres in v (A9). If v instantiates v_1, then v is a d-place quality if and only if v_1 is a d-place quality (A10). Further, the arity of a quality is determinate (A11).

A6. $\forall v \forall v_1 \ldots \forall v_d\, (Inheres(v_1, \ldots, v_d, v) \rightarrow Partic(v))$

A7. $\forall v \forall v_1 \ldots \forall v_d\, \forall w_1 \ldots \forall w_d\, ((Inheres(v_1, \ldots, v_d, v) \wedge Inheres(w_1, \ldots, w_d, v)) \rightarrow (v_1 = w_1 \wedge \ldots \wedge v_d = w_d))$

A8. $\forall v \forall v_1 \ldots \forall v_d \forall t((Inheres(v_1, \ldots, v_d, v) \wedge Exists(v, t)) \rightarrow (Subst(v_n) \wedge Partic(v_n) \wedge Exists(v_n, t)))$, for all n $(1 \leq n \leq d)$

A9. $\forall v \forall t\, (Subst(v) \wedge Partic(v) \wedge Exists(v, t) \rightarrow \exists v_1(Qua^1(v_1) \wedge Inheres(v, v_1) \wedge Exists(v_1, t)))$

A10. $\forall v\, \forall v_1(\exists t\, InstAt(v, v_1, t) \rightarrow (Qua^d(v) \leftrightarrow Qua^d(v_1)))$, for all $d \geq 1$.

A11. $\forall v\, (Qua^{d1}(v) \rightarrow \sim Qua^{d2}(v))$, where $d_1 \neq d_2$.

Excerpt from Fabian Neuhas, Pierre Grenon, Barry Smith, A Formal Theory of Substances, Qualities, and Universals. In Proceedings of the International Conference on Formal Ontology and Information Systems (2004)

Logic and Mathematics

Mathematicians, like philosophers, want to constuct valid arguments only.

Indeed there was an attempt in the early 20th century to constuct mathematics on the basis of logic alone.

$*54\cdot43.\ \vdash :.\ \alpha, \beta \in 1\ .\supset :\ \alpha \cap \beta = \Lambda\ .\equiv .\ \alpha \cup \beta \in 2$

Dem.

$\vdash .\ *54\cdot26\ .\supset \vdash :.\ \alpha = \iota`x\ .\ \beta = \iota`y\ .\supset :\ \alpha \cup \beta \in 2\ .\equiv .\ x \neq y\ .$

$[*51\cdot231] \qquad \equiv .\ \iota`x \cap \iota`y = \Lambda\ .$

$[*13\cdot12] \qquad \equiv .\ \alpha \cap \beta = \Lambda \qquad (1)$

$\vdash .\ (1)\ .\ *11\cdot11\cdot35\ .\supset$

$\vdash :.\ (\exists x, y)\ .\ \alpha = \iota`x\ .\ \beta = \iota`y\ .\supset :\ \alpha \cup \beta \in 2\ .\equiv .\ \alpha \cap \beta = \Lambda \qquad (2)$

$\vdash .\ (2)\ .\ *11\cdot54\ .\ *52\cdot1\ .\supset \vdash .$ Prop

From this proposition it will follow, when arithmetical addition has been defined, that $1+1=2$.

Excerpt from Alfred North Whitehead and Bertrand Russell, Principia Mathematica (1910)

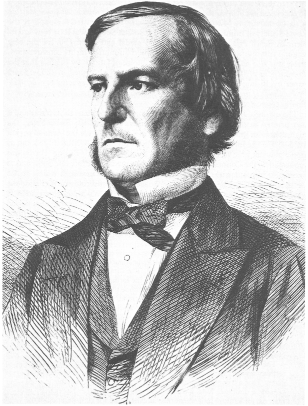
Logic and Mathematics

Although founding all mathematics on logic alone has proved impossible, logic is the basic working tool of all mathematicians.

But we can also look at logic as a branch of mathematics.

This insight came via George Boole in the 19th century.

In this course we assume some comfort with university level mathematics, e.g. from MATH1005/6005 or COMP1600/6260.



George Boole. c/o Wikimedia

Logic and Computer Science

In the modern world no discipline contributes more to logic than computer science.

- The electronic engineering of ‘logic gates’ in hardware;
- The data type of Boolean values in programming;
- Database queries;
- Formal specification of requirements and designs;
- Formal proof of the correctness of software and hardware;
- Computer assistance for mathematics (automated proving and proof checking);
- Artificial intelligence: machines that can reason;
- ...

Logic and Computer Science

This course gives an introduction to logic motivated by a computer science perspective.

- students from elsewhere - particularly philosophy and mathematics - very welcome!

We will need to spend our time introducing logic before much talk about applications.

- but we will make connections between logic and computing, particularly by focusing on how we model situations using the language of logic.
- more advanced courses like COMP4011 (Advanced Topics in Formal Methods and Programming Languages) and COMP3620/6320 (Artificial Intelligence), as well as the annual Logic Summer School, take this material further into applications.

Course Overview

Lectures

Three lectures per week

- except public holidays, and aiming to finish Wednesday of week 11.
- lectures recorded, and slides online in advance...
- *but* I will also do live 'whiteboard' work so in the lecture itself, preferably in person, is essential for the whole story.

My 'office hours' will be held outside the lecture theatre after each lecture.

Tutorials

One learns by doing, not by listening alone.

Weekly tutorials are for working through examples, with tutor assistance and demonstration.

- Not scheduled weeks 1 and 12
- Bring your own device, and paper to work on
- no software installation required; we will use the web-based Logic4Fun tool.

From week 3 there will be short tests at the start of each tutorial

- Cumulatively (best 7 of 9) worth 50% of final mark
- Some on paper, others online in Logic4Fun
- Each test will be similar to exercises completed in the previous week's tutorial
- You may only attempt the test in one tutorial per week!

Final Exam

The other 50% of your mark be an in-person on-paper final exam.

Open book (but no electronics)

- I am not primarily interested in your memorisation of logical rules...
- although memory does often help one work faster!

In recent years this course has not had an exam, but I will make sure that you have enough materials to adequately prepare.

Questions

A number of ways to ask questions...

- Interruptions during lectures permitted and encouraged!
- 'Office hours' with me after each lecture.
- The Ed discussion forum
  - Read pinned post and follow instructions e.g. keep post public except in very unusual circumstances, informative title;
  - Be bold and have a go at answering other students' questions!
  - Also staffed by tutors and myself
- Email: only where there are genuinely private considerations
  - To contact me please use comp2620@anu.edu.au rather than personal email - I will prioritise this address and bounce back course mails to my private address;
  - Mail tutors (addresses in Wattle) only about tutorial issues, including marking of tutorial tests.

Resources

The official class summaries are available on Programs and Courses: 2620, 6262. These provide substantially more detail than is in these slides and I strongly advise you to read it for this class, and all your other classes.

There is no compulsory reference for this course but my teaching is inspired in particular by:

- The online Logic Notes by John Slaney, which is highly recommended reading;
- Logic in Computer Science: Modelling and Reasoning about Systems, by Michael Huth and Mark Ryan

Where I draw from other sources I will acknowledge them in the relevant slides.

Call for Course Representatives

CSS Class Representatives

Class Student Representation is an important component of the teaching and learning quality assurance and quality improvement processes within the ANU College of Systems and Society (CSS).

Each semester, we put out a call for Class Representatives for all ANU College of Systems and Society (CSS) courses. Students can nominate themselves for one or more of the courses they are enrolled in.

Roles and Responsibilities

The role of Student Representatives is to provide ongoing constructive feedback on behalf of the student cohort to Course Conveners and to Associate Directors (Education) for continuous improvements to the course.

- Act as the official liaison between your peers and convenor.
- Be available and proactive in gathering feedback from your classmates.
- Attend regular meetings, and provide reports on course feedback to your course convenor
- Close the feedback loop by reporting back to the class the outcomes of your meetings.

Note: Class representatives will need to be comfortable with their contact details being made available via Wattle to all students in the class.
For more information regarding roles and responsibilities, contact: ANUSA CSS representatives (sa.cecc@anu.edu.au).

Why become a class representative?

- Ensure students have a voice to their course convener, lecturer, tutors, and College.
- Develop skills sought by employers, including interpersonal, dispute resolution, leadership and communication skills.
- Become empowered. Play an active role in determining the direction of your education.
- Become more aware of issues influencing your University and current issues in higher education.
- Course design and delivery. Help shape the delivery of your current courses, as well as future improvements for following years.

Want to be a class representative? Nominate today!

Please nominate yourself to comp2620@anu.edu.au by end of Week 2