COMP2620/6262 (Logic) Tutorial

Week 2

Semester 1, 2025

1. Recall the Backus-Naur form definition of the syntax of propositions:

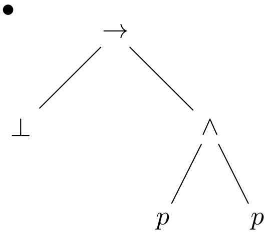
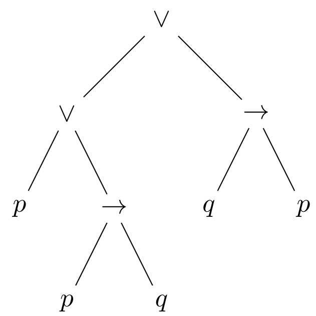
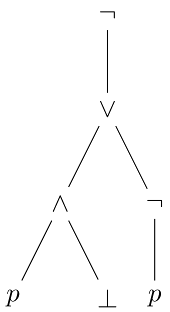
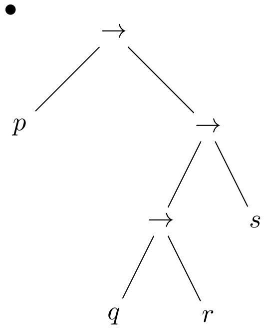
$$\varphi \quad := \quad p \mid \bot \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \to \varphi$$

   Draw trees showing how each of the following propositions, here expressed as strings, can be built with this definition.

   - $p \to (q \vee r)$
   - $(p \to q) \wedge r$
   - $\neg(\bot \wedge \neg(p \to \bot))$

2. Convert the following trees into string syntax. Avoid excessive parentheses using the rules

   - $\neg$ binds tighter than $\wedge$ and $\vee$, which in turn bind tighter than $\to$;
   - accept ambiguity when writing e.g. $p \wedge q \wedge r$ and $p \vee q \vee r$;
   - $\to$ is right associative.

   - 
   - 
   - 
   - 

3. **The test at the start of the next tutorial will resemble this question. Truth tables for connectives will be provided, as below. Rules for eliminating parentheses will not be provided. The propositions asked about will have 3 variables.**

   Recall the truth tables of propositional logic:

p	q	$\bot$	$\neg p$	$p \wedge q$	$p \vee q$	$p \to q$
1	1	0	0	1	1	1
1	0	“	“	0	1	0
0	1	“	1	0	1	1
0	0	“	“	0	0	1

For each proposition of question 1, and for each proposition you got as answers for question 2, construct their truth table. Are these propositions valid and/or satisfiable?

(You have not seen a truth table in lectures where there are 4 variables in play, so for the bottom right example you will need to think about how to set up the rows!)

4. Determine the validity of the following sequents using truth tables. Recall that you only need to consider the rows in which every premise has truth value 1.

    - $p \to \bot \vdash \neg p$
    - $p, \neg p \vdash q$
    - $p \to r, q \to r \vdash r \to (p \lor q)$
    - $(p \to q) \to p, \neg q \to \neg p \vdash q$

5. Some of our connectives can be defined in terms of some of the others; for example we could define $\neg p$ as $p \to \bot$, or $p \lor q$ as $\neg(\neg p \land \neg q)$.

    The Sheffer Stroke, also called nand, or 'not both', is a connective with truth table

p	q	$p \uparrow q$
1	1	0
1	0	1
0	1	1
0	0	1

    - How can we define $p \uparrow q$ using our usual connectives?
    - (Tricky) show that all our usual connectives can be defined using $\uparrow$ only. Hint: start by thinking about how you would define $\neg$.