COMP2620/6262 (Logic) Tutorial

Week 5

Semester 1, 2025

Tutorial Quiz

In each tutorial, apart from week 2, there is a short quiz on skills practised in the previous tutorial. Your top 7 quiz attempts, out of the 9 available, will collectively count for 50% of your final mark.

This week's quiz is on **Logic4Fun**. You should come prepared to use your device to log in to Logic4Fun, and have already joined our class, following instructions in Wattle. During the test you may look up Logic4Fun documentation on the Logic4Fun website: in particular you might like to have https://logic4fun.cecs.anu.edu.au/guide/built-ins open in a tab.

Your tutor will hand out paper face down with the description of the puzzle you need to translate into Logic4Fun syntax, along with some instructions and hints. On your tutor's signal you can turn your paper over and you have **ten minutes** to perform this task in the Solver window and submit. You may submit as often as you like, but later submissions overwrite earlier submissions, so do not submit at any time after your tutor has asked you to finish the task. This applies even if the test is still technically open on the website. For full marks your solution must pass the 'Check Syntax' button, and if your modelling is correct, the 'Solve' button will produce exactly one solution. Partial marks may still be available if these tests do not pass. You may ignore any warning that does not prevent the 'Check Syntax' button from printing 'Syntax checked...OK'.

Make sure that you know the number of the tutorial you are attending, and submit to that tutorial's number, even if it is different to your usual tutorial number.

This Week's Exercises

Recall the natural deduction rules for first order logic:

$$\frac{}{\varphi \vdash \varphi}\, A$$

$$\frac{\Gamma \vdash \varphi \quad \Gamma' \vdash \psi}{\Gamma, \Gamma' \vdash \varphi \wedge \psi}\, \wedge I \qquad \frac{\Gamma \vdash \varphi \wedge \psi}{\Gamma \vdash \varphi}\, \wedge E1 \qquad \frac{\Gamma \vdash \varphi \wedge \psi}{\Gamma \vdash \psi}\, \wedge E2$$

$$\frac{\Gamma, \varphi \vdash \psi}{\Gamma \vdash \varphi \to \psi}\, \to I \qquad \frac{\Gamma \vdash \varphi \to \psi \quad \Gamma' \vdash \varphi}{\Gamma, \Gamma' \vdash \psi}\, \to E$$

$$\frac{\Gamma, \varphi \vdash \bot}{\Gamma \vdash \neg\varphi}\, \neg I \qquad \frac{\Gamma \vdash \neg\varphi \quad \Gamma' \vdash \varphi}{\Gamma, \Gamma' \vdash \bot}\, \neg E \qquad \frac{\Gamma \vdash \bot}{\Gamma \vdash \varphi}\, \bot E$$

$$\frac{\Gamma \vdash \varphi}{\Gamma \vdash \varphi \vee \psi}\, \vee I1 \qquad \frac{\Gamma \vdash \psi}{\Gamma \vdash \varphi \vee \psi}\, \vee I2 \qquad \frac{\Gamma \vdash \varphi \vee \psi \quad \Gamma', \varphi \vdash \sigma \quad \Gamma'', \psi \vdash \sigma}{\Gamma, \Gamma', \Gamma'' \vdash \sigma}\, \vee E$$

$$\frac{\Gamma \vdash \neg\neg\varphi}{\Gamma \vdash \varphi}\, \neg\neg E$$

$$\frac{}{\vdash t = t}\, = I \qquad \frac{\Gamma \vdash t = u \quad \Gamma' \vdash \varphi[t/x]}{\Gamma, \Gamma' \vdash \varphi[u/x]}\, = E$$

$$\frac{\Gamma \vdash \varphi[t/x]}{\Gamma \vdash \exists x\varphi}\, \exists I \qquad a \notin FV(\Gamma, \varphi, \Gamma', \psi) : \frac{\Gamma \vdash \exists x\varphi \quad \Gamma', \varphi[a/x] \vdash \psi}{\Gamma, \Gamma' \vdash \psi}\, \exists E$$

$$a \notin FV(\Gamma, \varphi) : \frac{\Gamma \vdash \varphi[a/x]}{\Gamma \vdash \forall x\varphi}\, \forall I \qquad \frac{\Gamma \vdash \forall x\varphi}{\Gamma \vdash \varphi[t/x]}\, \forall E$$

1. We will do our most complex modelling into formal language with the Logic4Fun tool, but here are some sentences to translate into first order logic notation, to help you get used to that notation.

   For these translations, assume that the universe of discourse is some collection of people. L is a unary predicate meaning 'is a librarian', R is a unary predicate meaning 'is a reader', N is a binary predicate meaning its first argument 'needs' its second argument, and f is a unary function mapping each person to their favourite person.

   - All librarians are readers.
   - Every reader needs a librarian.
   - Some people's favourite person is themselves.
   - If anybody is a reader, then everybody needs librarians.
   - Everybody needs their favourite person, but not everybody's favourite person needs them.
   - If someone's favourite person is not a reader, then they must not be a librarian.

2. **The test at the start of the next tutorial will resemble this question. Natural deduction rules will be provided, as above. For full marks, you will need to use the five part notation taught in lectures: which premises are being used; a line number; a proposition; previous lines used; and the rule name.**

   Prove the following sequents by natural deduction.

   - $\forall x(Fx \to Gx),\ \forall x(Gx \to Hx) \ \vdash\ \forall x(Fx \to Hx)$
   - $\forall x(\neg Fx \to Gx) \ \vdash\ \forall x(\neg Gx \to Fx)$
   - $\forall x(Fx \to Gx),\ \exists x(Fx \wedge Hx) \ \vdash\ \exists x(Gx \wedge Hx)$
   - $\forall x(Fx \to \neg Gx) \ \vdash\ \neg\exists x(Fx \wedge Gx)$
   - $\forall x(Fx \to Hx),\ \forall x(Gx \to \neg Hx) \ \vdash\ \neg\exists x(Fx \wedge Gx)$
   - $\neg\forall x(Fx \to Gx) \ \vdash\ \exists x(Fx \wedge \neg Gx)$
   - $\vdash\ \exists x(Fx \to \forall y Fy)$ (Tricky)

3. Attempt the following puzzles from the Logic4Fun website.

   - Office Blocked
   - Library Books
   - Lost Property
   - Close Encounter