Australian National University

COMP3610/6361
Principles of Programming Languages

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Section 19

Concurrency

Concurrency and Distribution

so far we concentrated on semantics for sequential computation
but the world is not sequential. . .

- hardware is intrinsically parallel
- multi-processor architectures
- multi-threading (perhaps on a single processor)
- networked machines

Problems

aim: languages that can be used to model computations that execute in parallel and on distributed architectures

problems

- state-spaces explosion
  with n threads, each of which can be in 2 states, the system has 2^n states
- state-spaces become complex
- computation becomes nondeterministic
- competing for access to resources may deadlock or suffer starvation
- partial failure (of some processes, of some machines in a network, of some persistent storage devices)
- communication between different environments
- partial version change
- communication between administrative regions with partial trust (or, indeed, no trust)
- protection against malicious attack
- ...

Problems

this course can only scratch the surface

concurrency theory is a broad and active field for research

Process Calculi

- Observation (1970s): computers with shared-nothing architectures communicating by sending messages to each other would be important
  [Edsger W. Dijkstra, Tony Hoare, Robin Milner, and others]
- Hoare's Communicating Sequential Processes (CSP) is an early and highly-influential language that capture a message passing form of concurrency
- many languages have built on CSP including Milner's CCS and π-calculus, Petri nets, and others

IMP – Parallel Commands

we extend our while-language that is based on aexp, bexp and com

Syntax

$$\text{com} ::= \ldots \mid \text{com} \parallel \text{com}$$

Semantics

(par1) $$\frac{\langle c_0\ ,\ s\rangle \longrightarrow \langle c_0'\ ,\ s'\rangle}{\langle c_0 \parallel c_1\ ,\ s\rangle \longrightarrow \langle c_0' \parallel c_1\ ,\ s'\rangle}$$

(par2) $$\frac{\langle c_1\ ,\ s\rangle \longrightarrow \langle c_1'\ ,\ s'\rangle}{\langle c_0 \parallel c_1\ ,\ s\rangle \longrightarrow \langle c_0 \parallel c_1'\ ,\ s'\rangle}$$

IMP – Parallel Commands

Typing

(thread)

$$\frac{\Gamma \vdash c : \mathsf{unit}}{\Gamma \vdash c : \mathsf{proc}}$$

(par)

$$\frac{\Gamma \vdash c_0 : \mathsf{proc} \qquad \Gamma \vdash c_1 : \mathsf{proc}}{\Gamma \vdash c_0 \parallel c_1 : \mathsf{proc}}$$

Parallel Composition: Design Choices

- threads do not return a value
- threads do not have an identity
- termination of a thread cannot be observed within the language
- threads are not partitioned into ‘processes’ or machines
- threads cannot be killed externally

Asynchronous Execution

- semantics allow interleavings

$$\langle l := 1 \parallel l := 2\,,\ \{l \mapsto 0\}\rangle \longrightarrow \langle \textbf{skip} \parallel l := 2\,,\ \{l \mapsto 1\}\rangle \longrightarrow \langle \textbf{skip} \parallel \textbf{skip}\,,\ \{l \mapsto 2\}\rangle$$

$$\langle l := 1 \parallel l := 2\,,\ \{l \mapsto 0\}\rangle \longrightarrow \langle l := 1 \parallel \textbf{skip}\,,\ \{l \mapsto 2\}\rangle \longrightarrow \langle \textbf{skip} \parallel \textbf{skip}\,,\ \{l \mapsto 1\}\rangle$$

- assignments and dereferencing are *atomic*

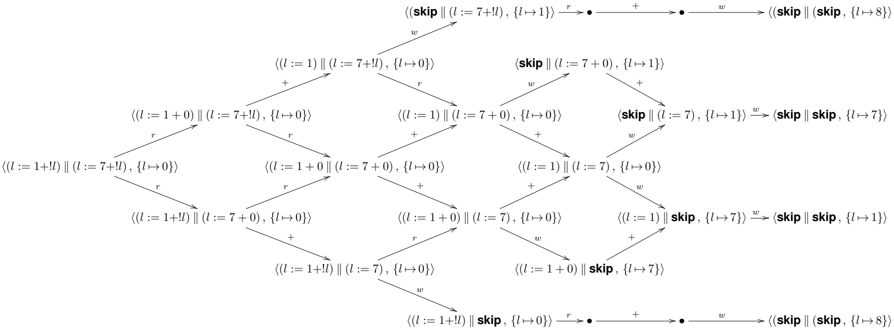
$$\langle l := N \parallel l := 2\,,\ \{l \mapsto 0\}\rangle \longrightarrow \langle \textbf{skip} \parallel l := 2\,,\ \{l \mapsto N\}\rangle \longrightarrow \langle \textbf{skip} \parallel \textbf{skip}\,,\ \{l \mapsto 2\}\rangle$$

$$\langle l := N \parallel l := 2\,,\ \{l \mapsto 0\}\rangle \longrightarrow \langle l := N \parallel \textbf{skip}\,,\ \{l \mapsto 2\}\rangle \longrightarrow \langle \textbf{skip} \parallel \textbf{skip}\,,\ \{l \mapsto N\}\rangle$$

for $N = 3498734590879238429384$.
(not something as the first word of one and the second word of the other)

Asynchronous Execution

- there interleaving in $(l := e) \parallel e'$



Morals

- combinatorial explosion
- drawing state-space diagrams only works for really tiny examples
- almost certainly the programmer does not want all those 3 outcomes to be possible
- complicated/impossible to analyse without formal methods

Parallel Commands – Nondeterminism

Semantics

(par1) $$\frac{\langle c_0\,,\,s\rangle \longrightarrow \langle c_0'\,,\,s'\rangle}{\langle c_0 \parallel c_1\,,\,s\rangle \longrightarrow \langle c_0' \parallel c_1\,,\,s'\rangle}$$

(par2) $$\frac{\langle c_1\,,\,s\rangle \longrightarrow \langle c_1'\,,\,s'\rangle}{\langle c_0 \parallel c_1\,,\,s\rangle \longrightarrow \langle c_0 \parallel c_1'\,,\,s'\rangle}$$

(+maybe rules for termination)

- study of nondeterminism
- $\parallel$ is not a partial function from state to state; big-step semantics needs adaptation
- can we achieve parallelism by nondeterministic interleaving
- communication via shared variable

Study of Parallelism (or Concurrency)

includes

Study of Nondeterminism

Dijkstra's Guarded Command Language (GCL)

- defined by Edsger Dijkstra for predicate transformer semantics
- combines programming concepts in a compact/abstract way
- simplicity allows correctness proofs
- closely related to Hoare logic

GCL – Syntax

- arithmetic expressions: aexp (as before)
- Boolean expressions: bexp (as before)
- Commands:

$$\begin{aligned} \mathsf{com} ::= {} & \textbf{skip} \mid \textbf{abort} \mid l := \mathsf{aexp} \mid \mathsf{com} \,;\, \mathsf{com} \mid \\ & \textbf{if}\ \mathsf{gc}\ \textbf{fi} \mid \textbf{do}\ \mathsf{gc}\ \textbf{od} \end{aligned}$$

- Guarded Commands:

$$\begin{aligned} \mathsf{gc} ::= {} & \mathsf{bexp} \to \mathsf{com} \mid \\ & \mathsf{gc} \mathbin{[\,]} \mathsf{gc} \end{aligned}$$

GCL – Semantics

- assume we have semantic rules for bexp and aexp (standard)
  we skip the deref-operator from now on
- assume a new configuration `fail`

Guarded Commands

(pos) $$\frac{\langle b\,,\ s\rangle \longrightarrow \langle \texttt{true}\,,\ s\rangle}{\langle b \rightarrow c\,,\ s\rangle \longrightarrow \langle c\,,\ s\rangle}$$

(neg) $$\frac{\langle b\,,\ s\rangle \longrightarrow \langle \texttt{false}\,,\ s\rangle}{\langle b \rightarrow c\,,\ s\rangle \longrightarrow \texttt{fail}}$$

(par1) $$\frac{\langle gc_0\,,\ s\rangle \longrightarrow \langle c\,,\ s'\rangle}{\langle gc_0 \ [\!]\ gc_1\,,\ s\rangle \longrightarrow \langle c\,,\ s'\rangle}$$

(par2) $$\frac{\langle gc_1\,,\ s\rangle \longrightarrow \langle c\,,\ s'\rangle}{\langle gc_0 \ [\!]\ gc_1\,,\ s\rangle \longrightarrow \langle c\,,\ s'\rangle}$$

(par3) $$\frac{\langle gc_0\,,\ s\rangle \longrightarrow \texttt{fail} \qquad \langle gc_1\,,\ s\rangle \longrightarrow \texttt{fail}}{\langle gc_0 \ [\!]\ gc_1\,,\ s\rangle \longrightarrow \texttt{fail}}$$

GCL – Semantics

Commands

- **skip** and sequencing ; as before (can drop determinacy)
- **abort** has no rules

(cond) $$\frac{\langle gc\,,\ s\rangle \longrightarrow \langle c\,,\ s'\rangle}{\langle \textbf{if}\ gc\ \textbf{fi}\,,\ s\rangle \longrightarrow \langle c\,,\ s'\rangle}$$

(loop1) $$\frac{\langle gc\,,\ s\rangle \longrightarrow \texttt{fail}}{\langle \textbf{do}\ gc\ \textbf{od}\,,\ s\rangle \longrightarrow \langle\!\langle s\rangle\!\rangle\ ^{\dagger}}$$

(loop2) $$\frac{\langle gc\,,\ s\rangle \longrightarrow \langle c\,,\ s'\rangle}{\langle \textbf{do}\ gc\ \textbf{od}\,,\ s\rangle \longrightarrow \langle c\,;\ \textbf{do}\ gc\ \textbf{od}\,,\ s'\rangle}$$

† new notation: behaves like **skip**

Processes

$$\textbf{do } b_1 \to c_1 \,[]\, \cdots \,[]\, b_n \to c_n \textbf{ od}$$

- form of (nondeterministically interleaved) parallel composition
- each c_i occurs atomically (uninterruptedly), provided b_i holds each time it starts

Some languages support/are based on GCL

- UNITY (Misra and Chandy)
- Hardware languages (Staunstrup)

GCL – Examples

- compute the maximum of x and y

if

$$x \geq y \rightarrow \max := x$$

$$[\!]$$

$$y \geq x \rightarrow \max := y$$

fi

- Euclid's algorithm

do

$$x > y \rightarrow x := x - y$$

$$[\!]$$

$$y > x \rightarrow y := y - x$$

od

GCL and Floyd-Hoare logic

guarded commands support a neat Hoare logic and decorated programs

Hoare triple for Euclid

$$\{x = m \wedge y = n \wedge m > 0 \wedge n > 0\}$$

Euclid

$$\{x = y = \mathrm{gcd}(m, n)\}$$

Proving Euclid's Algorithm Correct

- recall $\gcd(m,n)|m$, $\gcd(m,n)|n$ and

$$\ell|m,n \Rightarrow \ell|\gcd(m,n)$$

- invariant: $\gcd(m,n) = \gcd(x,y)$
- key properties:

$$\begin{aligned} \gcd(m,n) &= \gcd(m-n,n) && \text{if } m > n \\ \gcd(m,n) &= \gcd(m,n-m) && \text{if } n > m \\ \gcd(m,m) &= m \end{aligned}$$

Synchronised Communication

- communication by “handshake”
- possible exchange of value
  (localised to process-process (CSP) or to a channel (CCS))
- abstracts from the protocol underlying coordination
- invented by Hoare (CSP) and Milner (CCS)

Extending GCL

- allow processes to send and receive values on channels
  $\alpha!a$ evaluate expression a and send value on channel α
  $\alpha?x$ receive value on channel α and store it in x
- all interactions between parallel processes is by sending / receiving values on channels
- communication is synchronised (no broadcast yet)
- allow send and receive in commands c and in guards g:

$$\textbf{do } y < 100 \wedge \alpha?x \quad \rightarrow \quad \alpha!(x \cdot x) \parallel y := y + 1 \textbf{ od}$$

Extending GCL – Semantics

transitions may carry labels when possibility of interaction

$$\frac{}{\langle \alpha?x\,,\,s\rangle \xrightarrow{\alpha?n} \langle\!\langle s + \{x \mapsto n\}\rangle\!\rangle} \qquad \frac{\langle a\,,\,s\rangle \longrightarrow \langle n\,,\,s\rangle}{\langle \alpha!a\,,\,s\rangle \xrightarrow{\alpha!n} \langle\!\langle s\rangle\!\rangle}$$

$$\frac{\langle c_0\,,\,s\rangle \xrightarrow{\lambda} \langle c_0'\,,\,s'\rangle}{\langle c_0 \parallel c_1\,,\,s\rangle \xrightarrow{\lambda} \langle c_0' \parallel c_1\,,\,s'\rangle}$$ (+ symmetric)

$$\frac{\langle c_0\,,\,s\rangle \xrightarrow{\alpha?n} \langle c_0'\,,\,s'\rangle \qquad \langle c_1\,,\,s\rangle \xrightarrow{\alpha!n} \langle c_1'\,,\,s\rangle}{\langle c_0 \parallel c_1\,,\,s\rangle \longrightarrow \langle c_0' \parallel c_1'\,,\,s'\rangle}$$ (+ symmetric)

$$\frac{\langle c\,,\,s\rangle \xrightarrow{\lambda} \langle c'\,,\,s'\rangle}{\langle c\backslash\alpha\,,\,s\rangle \xrightarrow{\lambda} \langle c'\backslash\alpha\,,\,s'\rangle}\ \lambda \notin \{\alpha?n, \alpha!n\}$$

λ may be the empty label

Examples

- forwarder:

$$\textbf{do } \alpha?x \rightarrow \beta!x \textbf{ od}$$

- buffer of capacity 2:

$$\begin{array}{l} (\quad \textbf{do } \alpha?x \rightarrow \beta!x \textbf{ od} \\ \;\; \| \textbf{ do } \beta?x \rightarrow \gamma!x \textbf{ od })\backslash\beta \end{array}$$

External vs Internal Choice

the following two processes are *not* equivalent w.r.t. deadlock capabilities

$$\textbf{if } (\texttt{true} \wedge \alpha?x \rightarrow c_0) \;[]\; (\texttt{true} \wedge \beta?x \rightarrow c_1) \textbf{ fi}$$

$$\textbf{if } (\texttt{true} \rightarrow \alpha?x \,;\, c_0) \;[]\; (\texttt{true} \rightarrow \beta?x \,;\, c_1) \textbf{ fi}$$