$\{\} \vdash l := 5 ; !l : int$

$\langle l := 5 ; !l , \emptyset \rangle$

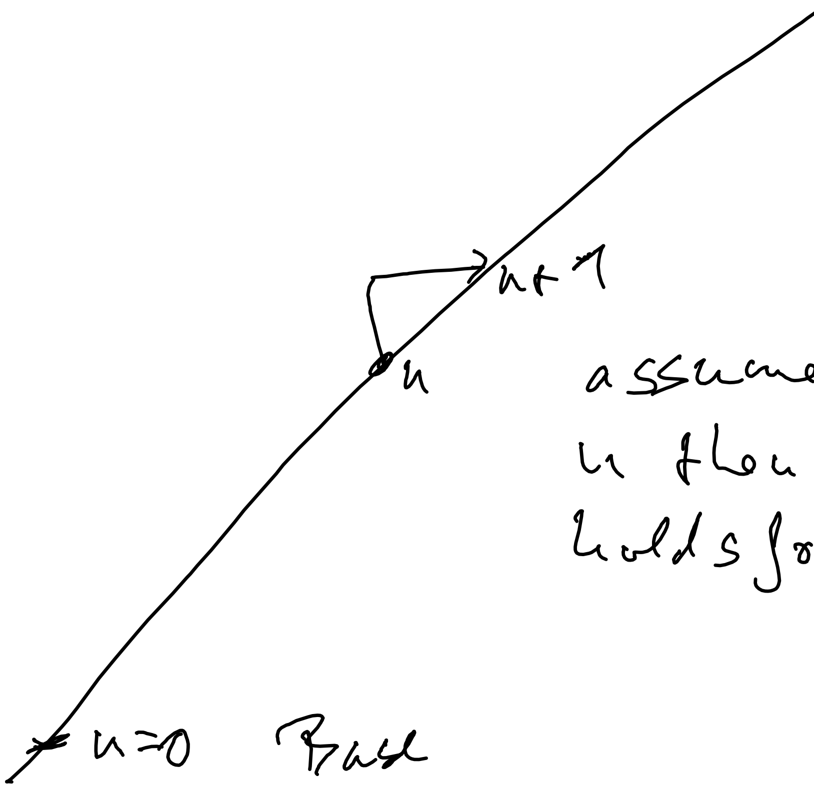
$\to \langle skip ; !l , \{l \mapsto 5\} \rangle$

$\to \langle !l , \{l \mapsto 5\} \rangle$

$\{\} \vdash !l : int$

Fix

assign1 $\langle l := n , s \rangle \to \langle skip , s + \{l \mapsto n\} \rangle$
if $l \in dom(s)$



assume property for n then show property holds for n+1

$\phi(n): 0 + \dots n = \frac{n \cdot (n+1)}{2}$

Base Case: $\phi(0): 0 = \frac{0 \cdot (0+1)}{2}$ ✓

Step: Assum $\phi(k)$:

$$0 + \dots + k = \frac{k \cdot (k+1)}{2}$$

we should show $\phi(k+1)$

$$0 + \dots + k + (k+1) = \frac{(k+1)((k+1)+1)}{2}$$

$$0 + \dots + k + (k+1) \overset{I.H.}{=} \frac{k \cdot (k+1)}{2} + k + 1 = \frac{k \cdot (k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

Base case

$\forall n.\ \phi(n)$

Show that

$\forall s, E', s', E'', s''.$

IF $\langle n, s\rangle \to \langle E', s'\rangle$ and

$\langle n, s\rangle \to \langle E'', s''\rangle$

then

$\langle E', s'\rangle = \langle E'', s''\rangle$ ✓

Induction step

for +

Assume: $\phi(E_1)$ $\forall s, s', s'' E', E''$

$(E_1, s) \rightarrow (E', s')$ and

$\langle E_1, s) \rightarrow (E'', s'')$ then

$\langle E', s'\rangle = \langle E'', s''\rangle$

$\phi(E_2)$

Show

$\phi(E_1 + E_2)$

Assume $\langle E_1 + E_2, s\rangle \rightarrow \langle E', s'\rangle$

and $\langle E_1 + E_2\ s\rangle \rightarrow \langle E'', s'\rangle$

Look at semantics

$$\frac{\langle E_1, s\rangle \to \langle E_1', s'\rangle}{\langle E_1 + E_2, s\rangle \to \langle E_1' + E_2, s'\rangle}$$

$$\frac{\langle E_2, s\rangle \to \langle E_2', s'\rangle}{\langle v + E_2, s\rangle \to \langle v + E_2', s\rangle}$$

$$\langle n_1 + n_2, s\rangle \to \langle n, s\rangle \quad \text{if } n = n_1 + n_2$$

Case distinction

$E_1 = n_1$ and $E_2 = n_2$ only 1 successor ✓

Case

$E_1 = n$ E_2 is expression.

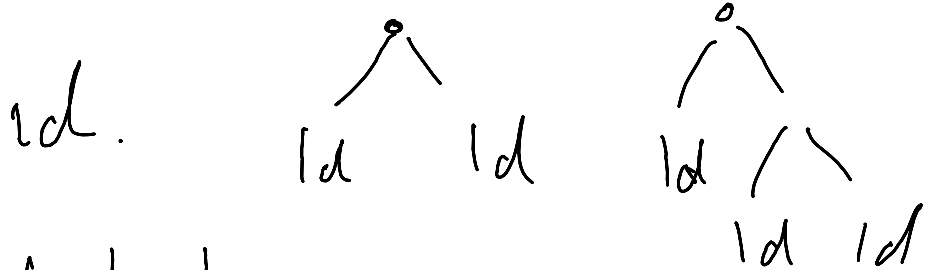
$$\frac{\langle E_2, s\rangle \to \langle E_2', s'\rangle}{\langle n + E_2, s\rangle \to \langle n + E_2', s'\rangle}$$

unique by assumption

Data Structure

BTree

$BT ::= ld \mid BT\ BT$



We want to show

$\phi(t)$ for all trees t

Base case: $\phi(ld)$

Step: $\phi(T_1) \wedge \phi(T_2) \Rightarrow \phi(T_1\ T_2)$

$\phi(E) \stackrel{def}{=} \forall s, E', s', E'', s''.$

$\quad$ if $\langle E, s\rangle \to \langle E', s'\rangle \wedge \langle E, s\rangle \to \langle E'', s''\rangle$

$\quad$ then $\langle E', s'\rangle = \langle E'', s''\rangle$

Assume $\phi(E_1), \phi(E_2), \phi(E_2)$

we want to show $\phi(\text{if } E_1 \ldots)$

$\forall s, E', s', E'', s''.$

$\quad$ if $\langle \text{if } E_1 \text{ then } E_2 \text{ else } E_3, s\rangle \to \langle E', s'\rangle$

$\quad$ and $\langle \text{if } E_1 \text{ then } E_2 \text{ else } E_3, s\rangle \to \langle E'', s''\rangle$

then show $\langle E', s'\rangle = \langle E'', s'\rangle$

Case $E_1 = \text{true}$ then

$$\langle \text{if } E_1 \text{ then } E_2 \text{ else } E_3, s\rangle \to \langle E_2, s\rangle$$

Case $E_1 = \text{false}$

case $E_1 \neq \text{true}, E_1 \neq \text{false}$

$$\frac{\langle E_1, s\rangle \to \langle E_1', s'\rangle}{\langle \text{if } E_1 \text{ then } E_2 \text{ else } E_3, s\rangle \to \langle \text{if } E_1' \text{ then } E_2 \text{ else } E_3, s'\rangle}$$

we have $\cancel{\phi}(E_1)$