COMP3610/6361

28/09/2023

$\{l_0 = n \wedge n > 0\}\ l_1 := 1\ \underbrace{\{l_0 = n \wedge n > 0 \wedge l_1 = 1\}}$

is this a good predicate?

Q.

$\{l_0 = n \wedge n > 0 \wedge l_1 = 1 \wedge l_0 > 0\}$

$l_1 := !l_1 \cdot !l_0;$

$l_0 := !l_0 - 1$

$\{l_0 = n \wedge n > 0 \wedge l_1 = 1\}$

does not hold if $n > 1$

Find loop invariant

(i) $\{P \wedge l_0 > 0\}\ l_1 := !l_1 \cdot !l_0;\ l_0 := !l_0 - 1\ \{P\}$

(ia)

(ii) $\{P_1 \wedge l_0 \leq 0\} \Longrightarrow \{l_1 = n!\}$

(iii) P can only include l_0, l_1, n

(iv) P must talk about l_1

P? $l_1 \cdot l_0! = n! \wedge l_0 \geq 0$

Let's denote l_0, l_1 after execution by $\vec{l_0}, \vec{l_1}$

we want to show $\vec{l_1} \cdot \vec{l_0}! = n!$

that is equivalent to $(l_1 \cdot l_0)(l_0 - 1)! = n!$

$\Leftrightarrow l_1 \cdot (l_0 \cdot (l_0 - 1)! = n!$

$\Leftrightarrow l_1 \cdot l_0! = n!$

true by precondition

"run the program"

1st loop in

$l_1 = 1 \cdot n$

$l_0 = n-1$

2nd

$l_1 = n \cdot n-1$

$l_0 = n-2$

3rd

$l_1 = n \cdot n-1 \cdot n-2$

$l_2 = n-3$

$$l_1 \cdot l_0! = n!$$

$\{l_0 = n \wedge n > 0\}\ l_1 := 1\ \{l_0 = n \wedge n \geq 0 \wedge l_1 = 1\}$

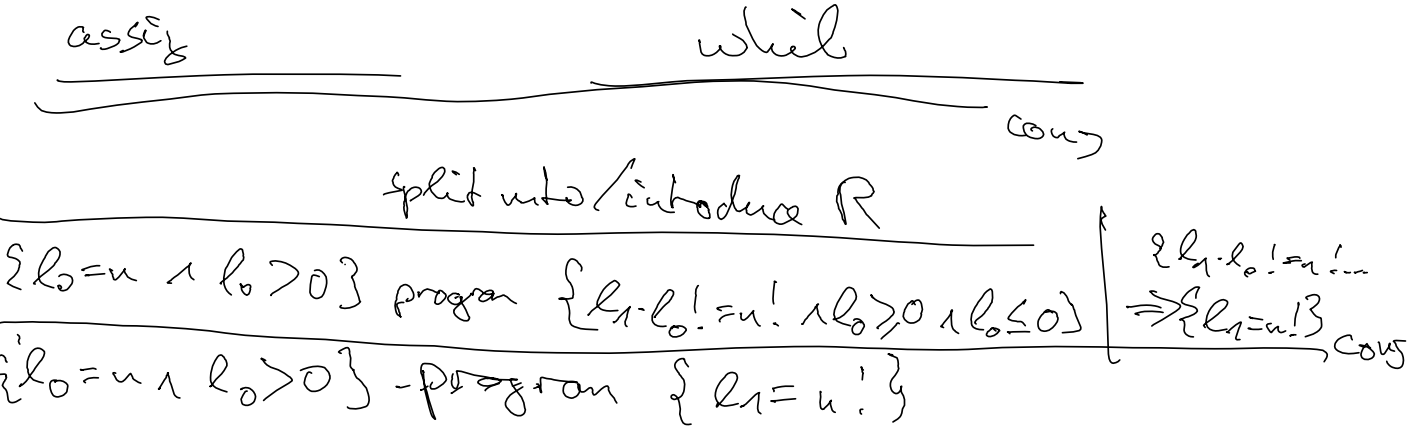
$\{l_1 \cdot l_0! = n! \wedge l_0 \geq 0\}$ while .... $\{l_1 \cdot l_0! = n! \wedge l_0 \geq 0 \wedge l_0 \leq 0\}$

$= \{l_1 \cdot l_0! = n! \wedge l_0 = 0\}$

$\{l_1 \cdot l_0! = n! \wedge l_0 = 0\} \Rightarrow \{l_1 = n!\}$

post of while loop

post condition of program.


assign
while
conj
split into / introduce R
{l0 = n ∧ l0 > 0} program {l1 · l0! = n! ∧ l0 ≥ 0 ∧ l0 ≤ 0}
{l1 · l0! = n! ∧ ... ⇒ {l1 = n!}
conj
{l0 = n ∧ l0 > 0} - program {l1 = n!}