COMP 3610

11/10/2023

if _ then _ else _

while _ do _

$\ell := 3;\ \mathbf{if}\ (!\ell \geq 2 \rightarrow \ell := 5)\ [\,]\ (!\ell \leq 6 \rightarrow \ell := 6\ [\,]\ !\ell < 3 \rightarrow \ell := 0)$
$\mathbf{fi}$

we know $\langle !\ell \geq 2 \rightarrow \ell := 5, s \rangle \longrightarrow \langle \ell := 5, s \rangle$ (pos)

$\langle !\ell \leq 6 \rightarrow \ell := 6, s \rangle \longrightarrow \langle \ell := 6, s \rangle$ (pos)

$\langle !\ell < 3 \rightarrow \ell := 0, s \rangle \longrightarrow \mathrm{fail}$ (neg)

$$l := 3;\ \text{if}\ (!l \geq 2 \rightarrow l := 5)\ [\,]\ (!l \leq 6 \rightarrow l := 6\ \ [\,]\ !l < 3 \rightarrow l := 0)$$

$$\text{fi}$$

$$\rightarrow^{*} (\text{skip},\ s + \{l \mapsto 5\})$$

$$\rightarrow^{*} (\text{skip},\ s + \{l \mapsto 6\})$$

$\text{if } b \text{ then } c_1 \text{ else } e_2$

$\widehat{=}$

if

$b \rightarrow c_1$

$[]$

$\neg b \rightarrow c_2$

fi

while b do c

$\hat{=}$ do

$b \rightarrow c$

od

Exercis3

max (x, y)

if

$x \geq y \rightarrow x$

[]

$y \geq x \rightarrow y$

fi

$x = 50$

$y = 15$

$\rightarrow$

$x = 35$

$y = 15$

$\rightarrow x = 20$

$y = 15$

$\rightarrow x = 5$

$y = 15$

$\rightarrow x = 5$

$y = 10$

$\rightarrow x = 5$

$y = 5$

$$\underline{\gcd(m-n, n)} \mid m$$

$$\gcd(m-n, n) \mid n \checkmark$$

$$\ell \mid m, n \quad \Rightarrow \ell \mid \gcd(m-n, n)$$

$$\gcd(m-n, n) \mid m-n \qquad \exists \ell: x \cdot \ell = m-n$$

$$\underbrace{\gcd(m-n, n)}_{x} \mid n \qquad \exists k: \underline{x \cdot k = n}$$

$$\underbrace{x \cdot \ell + x \cdot k}_{= x \cdot (\ell + k)} = m - n + n = m$$

$$\ell \mid m, n \quad \Rightarrow \ell \mid \gcd(m-n, n)$$

$\exists k_1 \quad \ell \cdot k_1 = m$?

$\exists k_2 \quad \ell \cdot k_2 = n$

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we have to show

$$\forall \ell.\ \ell \mid m, n \Rightarrow \ell \mid \gcd(m-n, n)$$

we know

$$\forall.\ \ell \mid m-n, n \Rightarrow \ell \mid \gcd(m-n, n)$$

let's assume $\ell \mid m, n$

$\Leftrightarrow \exists x_1, x_2.\ \ \ell \cdot x_1 = m \quad \Rightarrow \ell \cdot x_1 - \ell \cdot x_2 = m-n$

$x_1 \geq x_2 \qquad \ell \cdot x_2 = n$

$\ell(x_1 - x_2) = m-n$

$\Leftrightarrow \ell \mid m-n$

$$(\alpha!5 \parallel \ell:=3) \parallel \underline{\underline{\alpha?x}}$$

$$\alpha?x \xrightarrow{\alpha?5} \langle\langle s + \{x \mapsto 5\}\rangle\rangle$$

$$\frac{}{\langle \alpha!5 \parallel \ell:=3, s\rangle \xrightarrow{\alpha!5} \langle \dots \rangle}$$

$$(\text{do } \alpha?x \to \beta!x \text{ od}$$
$$\| \text{ do } \beta?x \to \gamma!x \text{ od}) \setminus \beta$$

$$\| \beta?x$$

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by semantics of gc, do

$$\text{do } \alpha?x \to \beta!x \text{ od} \xrightarrow{\beta!n} \ldots$$

hence by ParLeft

$$\text{do} \ldots \text{od} \| \text{do } \beta?x \to \gamma!x \text{ od} \xrightarrow{\beta!n}$$

but by restriction operator $\beta!n$

$$(\text{do} \ldots \text{od} \| \text{do} \ldots \text{od}) \setminus \beta \not\rightarrow$$

$$\mathit{Max}(x,y) \stackrel{def}{=} x > y \rightarrow \alpha!x \rightarrow nil$$
$$+\ y \geq x \rightarrow \alpha!y \rightarrow nil$$

$$P() \stackrel{def}{=} \alpha?x \rightarrow \beta?y \rightarrow \mathit{Max}(x,y)$$

$$\overline{(\alpha?x \rightarrow \beta!x \rightarrow nil) \xrightarrow{\alpha?n} (\beta!n \rightarrow nil)}$$