

# COMP3610/6361

## Principles of Programming Languages

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# Section 1

## Pure CCS

## Towards a more basic language

**aim:** removal of variables to reveal symmetry of input and output

- transitions for value-passing carry labels  $\tau$ ,  $a?n$ ,  $a!n$

$$\begin{array}{ccc}
 \alpha?x \rightarrow p & \xrightarrow{\alpha?0} & p[0/x] \\
 & \searrow^{\alpha?n} & \vdots \\
 & & p[n/x]
 \end{array}$$

- this suggests introducing *prefix*  $\alpha?n.p$  (as well as  $\alpha!n.p$ ) and *view*  $\alpha?x \rightarrow p$  as a (*infinite*) *sum*  $\sum_n \alpha?n.p[n/x]$
- view*  $\alpha?n$  and  $\alpha!n$  as *complementary* actions
- synchronisation can only occur on complementary actions

## Pure CCS

- Actions:  $a, b, c, \dots$
- Complementary actions:  $\bar{a}, \bar{b}, \bar{c}, \dots$
- Internal action:  $\tau$
- Notational convention:  $\bar{\bar{a}} = a$

- Processes:

$p ::= \lambda.p$	prefix	$\lambda$ ranges over $\tau, a, \bar{a}$ for any action
$\sum_{i \in I} p_i$	sum	$I$ is an index set
$p_0 \parallel p_1$	parallel	
$p \setminus L$	restriction	$L$ a set of actions
$p[f]$	relabelling	$f$ a relabelling function on actions
$P$		process identifier

- Process definitions:

$$P \stackrel{\text{def}}{=} p$$

# Pure CCS – Semantics

## Guarded processes (prefixing)

$$\lambda.p \xrightarrow{\lambda} p$$

### Sum

$$\frac{p_j \xrightarrow{\lambda} p'}{\sum_{i \in I} p_i \xrightarrow{\lambda} p'} \quad j \in I$$

### Parallel composition

$$\frac{p_0 \xrightarrow{\lambda} p'_0}{p_0 \parallel p_1 \xrightarrow{\lambda} p'_0 \parallel p_1} \qquad \frac{p_1 \xrightarrow{\lambda} p'_1}{p_0 \parallel p_1 \xrightarrow{\lambda} p_0 \parallel p'_1}$$

$$\frac{p_0 \xrightarrow{a} p'_0 \quad p_1 \xrightarrow{\bar{a}} p'_1}{p_0 \parallel p_1 \xrightarrow{\tau} p'_0 \parallel p'_1}$$

## Pure CCS – Semantics

### Restriction

$$\frac{p \xrightarrow{\lambda} p'}{p \setminus L \xrightarrow{\lambda} p' \setminus L} \quad \lambda \notin L \cup \bar{L}$$

where  $\bar{L} = \{\bar{a} \mid a \in L\}$

### Relabelling

$$\frac{p \xrightarrow{\lambda} p'}{p[f] \xrightarrow{\lambda} p'[f]}$$

where  $f$  is a function such that  $f(\tau) = \tau$  and  $f(\bar{a}) = \overline{f(a)}$

### Identifiers

$$\frac{p \xrightarrow{\lambda} p'}{P \xrightarrow{\lambda} p'} \quad P \stackrel{\text{def}}{=} p$$

## From Value-passing to Pure CCS

translation from a value-passing CCS *closed* term  $p$  to a pure CCS term  $\widehat{p}$

$p$	$\widehat{p}$	
<b>nil</b>	<b>nil</b>	
$(\tau \rightarrow p)$	$\tau.\widehat{p}$	
$(\alpha!a \rightarrow p)$	$\overline{\alpha}m.\widehat{p}$	where $a$ evaluates to $m$
$(\alpha?x \rightarrow p)$	$\sum_{m \in \text{int}} \alpha m.\widehat{p[m/x]}$	
$(b \rightarrow p)$	$\widehat{p}$ <b>nil</b>	if $b$ evaluates to true if $b$ evaluates to false
$p_0 + p_1$	$\widehat{p}_0 + \widehat{p}_1$	
$p_0 \parallel p_1$	$\widehat{p}_0 \parallel \widehat{p}_1$	
$p \setminus L$	$\widehat{p} \setminus \{\alpha m \mid \alpha \in L \wedge m \in \text{int}\}$	
$P(a_1, \dots, a_k)$	$P_{m_1, \dots, m_k}$	where $a_i$ evaluates to $m_i$

For every definition  $P(x_1, \dots, x_k)$  we have a collection of definitions  $P_{m_1, \dots, m_k}$  indexed by  $m_1, \dots, m_k \in \text{int}$

# Correspondence

## Theorem

$$p \xrightarrow{\lambda} p' \quad \text{iff} \quad \widehat{p} \xrightarrow{\widehat{\lambda}} \widehat{p}'$$