

# COMP 3610 Tutorial 7

27 September, 2023

## Exercise 1

Below we define a BNF specification (and abstract syntax) of the language of Roman numerals less than five hundred.

$$\begin{aligned} \text{Roman} & ::= \text{Hundreds Tens Units} \\ \text{Hundreds} & ::= \epsilon \mid \mathbf{C} \mid \mathbf{CC} \mid \mathbf{CCC} \mid \mathbf{CD} \\ \text{Tens} & ::= \text{LowTens} \mid \mathbf{XL} \mid \mathbf{L} \text{LowTens} \mid \mathbf{XC} \\ \text{LowTens} & ::= \epsilon \mid \text{LowTens} \mathbf{X} \\ \text{Units} & ::= \text{LowUnits} \mid \mathbf{IV} \mid \mathbf{V} \text{LowUnits} \mid \mathbf{IX} \\ \text{LowUnits} & ::= \epsilon \mid \text{LowUnits} \mathbf{I} \end{aligned}$$

The language of Roman numerals is subject to context constraints that the number of X's in LowTens and I's in LowUnits can be no more than three. Remember  $\epsilon$  represents the empty string. Provide semantic functions and semantic equations for a denotational definition of Roman numerals that furnishes the numeric value of each string in the language. Assume that the context constraints have been verified by other means.

## Exercise 2

Consider our simple IMP language (no functions), extended by the data type of *products* – see Section 8 of the lecture. Give a denotational semantics for that extension.

## Exercise 3

Give a counterexample for the following assignment rule in axiomatic semantics (showing them to be unsound):

- $\vdash \{P[l/a]\} l := a \{P\}$
- $\vdash \{\mathbf{true}\} l := a \{l = a\}$
- $$\frac{\{P\} c \{Q'\} \quad \models Q \Rightarrow Q'}{\{P\} c \{Q\}}$$