The Australian National University

First Semester Examination May 27, 2022

Theory of Computation

Final Exam

Writing Period: 150 minutes
Study Period: 15 minutes
Permitted Materials: None

Student Number

Please read the following instructions carefully.

- This is a *closed book exam.*
  No material other than the provided scratch paper is permitted.
- Use either a black or a blue pen. Use the space provided.
  Marks may be lost for giving information that is irrelevant.
- Please write your student number on *every sheet of paper.*
- This exam consists of 7 questions worth a total of 150 points
  (about 1 point per minute).
- This examination paper is *confidential* and is not to be taken from the examination room.

For official use only:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total
40	25	15	15	20	10	25	150

Question 1 **True or False - With Justification** (40 points)

Please read each of the following statements *very carefully*, some of them are trying to trick you. For each of the statements, state whether it is true or false and justify your answer in 1–2 sentences. Each correct answer gives 1 point and each correct justification gives 3 additional points.

(a) The union of infinitely many regular languages is regular.

(b) For a regular language L containing exactly n strings, there is a nondeterministic finite automaton with at most n states that accepts L.

(c) If a language fulfills the condition of the pumping lemma, then it is regular.

(d) If a context-free grammar has no unit productions and no ϵ-productions, then for any given string, the height of all parse trees of that string is bounded.

(e) The language $L = \{(M, w) \mid M \text{ does not accept } w\}$ is recursively enumerable.

(f) If a language L is recursively enumerable and its complement L^c is recursively enumerable, then L is decidable.

(g) There are undecidable languages over the alphabet containing only one symbol.

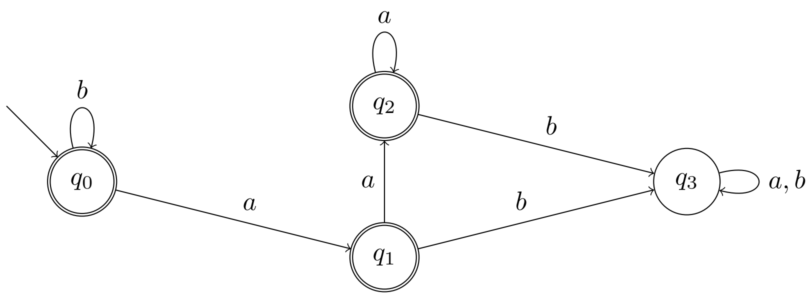
(h) P is known to be closed under intersection.

(i) NP is the set of all problems that do not have a polynomial-time algorithm.

(j) If L_1 is NP-complete, L_2 is NP and $L_1 \leq_P L_2$, then L_2 is NP-complete.

Question 2 **Finite Automata and Regular Expressions** 5 + 10 + 10 points

Consider the following finite automaton A over the alphabet $\Sigma = \{a, b, c\}$.



(a) Is A deterministic? If not, convert A into a DFA.

(b) Is A minimal? If not, convert A into a minimal DFA.

(c) Convert A into a regular expression.

Question 3 **Decidability** 20 points

Consider the language

$$L = \{M \mid M \text{ is a Turing machine that has at least one unreachable state }\}.$$

Prove that L is undecidable.

Question 4 **NP-Completeness** 10 points

Explain in 3–4 clear English sentences why NP-complete problems are considered intractable.

Question 5 **Proving NP-Completeness** 5 + 20 points

Consider the *subgraph-isomorphism problem*: given two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, does G_1 contain a copy of G_2 as a subgraph? I.e., is there a subset $U \subseteq V_1$ and a function $i : U \to V_2$ such that

$$(v_1, v_2) \in E_1 \cap (U \times U) \iff (i(v_1), i(v_2)) \in E_2?$$

(a) Prove that the subgraph-isomorphism problem is in NP.

(b) The *k-clique problem* is the question whether a given graph $G = (V, E)$ has a subset of vertices $C \subseteq V$ such that C has at least k elements and every two vertices in C are connected with an edge in G.

Show that the clique problem can be reduced to the subgraph-isomorphism problem: Give a polynomial-time computable function f such that G has a k-clique if and only if G_2 is a subgraph of G_1 for $(G_1, G_2) = f(G)$. (*Hint: choose G_2 to be the complete graph with k vertices.*)

Question 6 **PSPACE = NPSPACE vs P = NP** 10 points

Explain in 3–4 clear English sentences why P $\stackrel{?}{=}$ NP question is more difficult to decide than PSPACE $\stackrel{?}{=}$ NPSPACE?

Question 7 **Error Reduction of RP** 25 points

Let $L \subseteq \Sigma^*$ be such that there exists a polynomial-time probabilistic Turing machine (PTM) M satisfying for every $w \in \Sigma^*$: (1) If $w \in L$, then $Pr[M \text{ accepts } w] \geq n^{-c}$ and (2) if $w \notin L$, then $Pr[M \text{ accepts } w] = 0$.

Prove that for every $d > 0$ there exists a polynomial-time PTM M' such that for every $w \in \Sigma^*$, **(1)** if $w \in L$ then $Pr[M \text{ accepts } w] \geq 1 - 2^{-n^d}$, and **(2)** if $w \notin L$ then $Pr[M \text{ accepts } w] = 0$.