ANU COMP3630: Final Exam 2020

May 27, 2022

1 General

For each of the statements below, determine whether it is true or false, and justify your answer in at most two sentences.

1. If A is a DFA with n states and it accepts all strings of length $< n$, then A accepts all strings.
2. The language $L = \{ww | w \text{ is an element of } \{a, b\}^*\}$ is regular.
3. For regular expressions r and s, we have that $L((r|s)^*) \subseteq L(r^*|s^*)$.
4. To derive a string of length n in a grammar in Chomsky normal form, one needs to use exactly $2n - 1$ productions.
5. If P is a PDA where no transition changes the number of symbols on the stack, then the language that P accepts by final state is regular.
6. The grammar $S \to bSb \mid A \quad A \to aA \mid b$ is ambiguous.
7. If a language L is the union of two undecidable languages, then L is undecidable itself.
8. It is decidable whether a Turing machine accepts the empty string.
9. Given an example of a language L_1 and a Language L_2 such that
   - L_1 is not NP-complete, but L_2 is NP-complete
   - The union of L_1 and L_2 is not NP-complete
   - The intersection of L_1 and L_2 is NP-complete.
10. If the language L is in NP, then the complement of L is in PSPACE.

2 Finite Automata and Regular Languages (15 credits)

Let L be a regular language. Show that the set of all strings $w \in L$ that are of odd length is also a regular language. Does the same also hold for the strings of even length?

3 Context Free Languages and Pushdown Automata (20 credits)

Consider the language $L = \{wcx \mid w, x \in \{a, b\}^*, w \neq x\}$ over the alphabet $\Sigma = \{a, b, c\}$. Is the language L context free? Either give a proof of L being context free, or a proof of L not being context free.

4 Turing Machines and Recursive Languages (15 credits)

A useless state of a deterministic Turing machine M is a state q that the machine never enters on any input. In other words, q is useless if for all strings w, running M on input w never causes the TM to enter state q.

Show that the problem of determining whether a (deterministic) TM M has a useless state is undecidable.

5 Complexity (20 credits)

Let TWO be the problem of deciding whether a boolean formula in 3-CNF has at least two different variable assignments under which it evaluates to true.

Show that TWO is NP-complete.