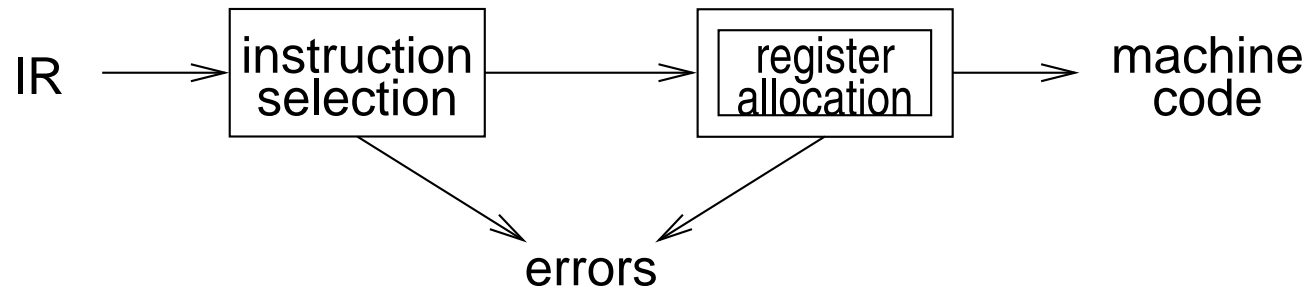


Register Allocation

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Register allocation



Register allocation:

- have value in a register when used
- limited resources
- changes instruction choices
- can move loads and stores
- optimal allocation is difficult
 - ⇒ NP-complete for $k \geq 1$ registers

Register allocation by simplification

Assume K registers

1. *Build* interference graph G : for each program point
 - (a) compute set of temporaries simultaneously live
 - (b) add edge to graph for each pair in set
2. *Simplify*: Color graph using a simple heuristic
 - (a) suppose G has node m with degree $< K$
 - (b) if $G' = G - \{m\}$ can be colored then so can G , since nodes adjacent to m have at most $K - 1$ colors
 - (c) each such simplification will reduce degree of remaining nodes leading to more opportunity for simplification
 - (d) leads to recursive coloring algorithm
3. *Spill*: suppose $\nexists m$ of degree $< K$
 - (a) target some node (temporary) for spilling (optimistically, spilling node will allow coloring of remaining nodes)
 - (b) remove and continue simplifying

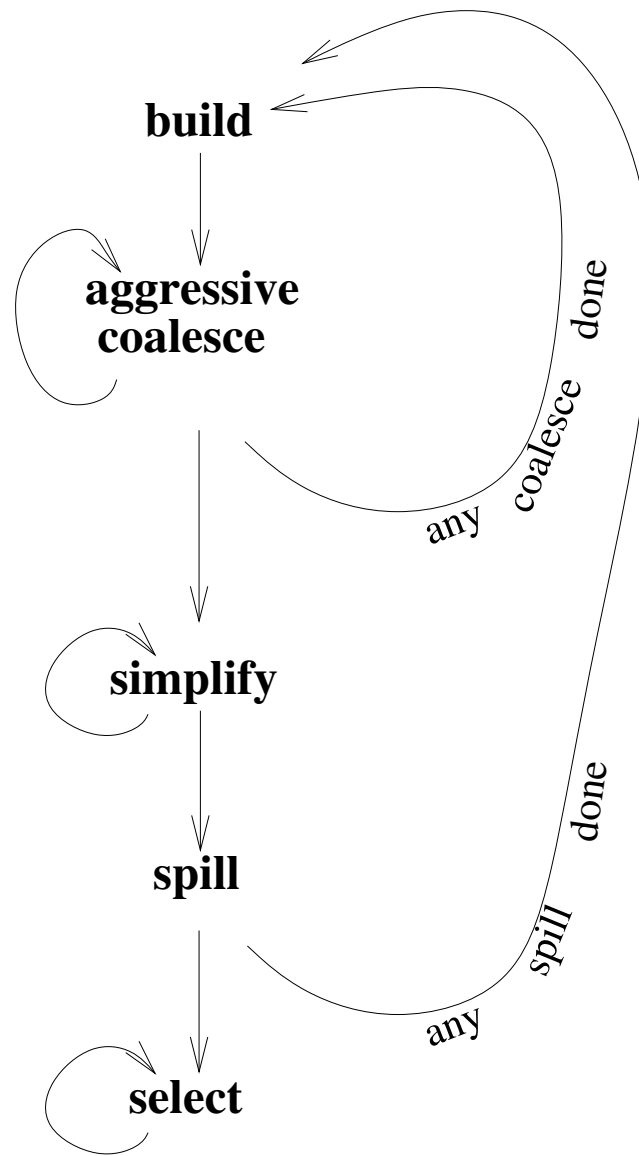
Register allocation by simplification (cont.)

4. *Select*: assign colors to nodes
 - (a) start with empty graph
 - (b) must be a color for non-spill nodes (basis for removal)
 - (c) if adding spill node and no color available (neighbors already K-colored) then mark as an *actual spill*
 - (d) repeat select
5. *Start over*: if select has no actual spills then finished, otherwise
 - (a) rewrite program to fetch actual spills before each use and store after each definition
 - (b) recalculate liveness and repeat

Coalescing

- Can delete a *move* instruction when source s and destination d do not interfere:
 - *coalesce* them into a new node whose edges are the union of those of s and d
- In principle, any pair of non-interfering nodes can be coalesced
 - unfortunately, the union is more constrained and new graph may no longer be K -colorable
 - overly aggressive

Simplification with aggressive coalescing



Conservative coalescing

Apply tests for coalescing that preserve colorability.

Suppose a and b are candidates for coalescing into node ab .

Briggs: coalesce only if ab has $< K$ neighbors of *significant* degree $\geq K$

- *simplify* first removes all insignificant-degree neighbors
- ab will then be adjacent to $< K$ neighbors
- *simplify* can then remove ab

George: coalesce only if all significant-degree neighbors of a already interfere with b

- *simplify* removes all insignificant-degree neighbors of a
- remaining significant-degree neighbors of a already interfere with b so coalescing does not increase the degree of any node

Iterated register coalescing

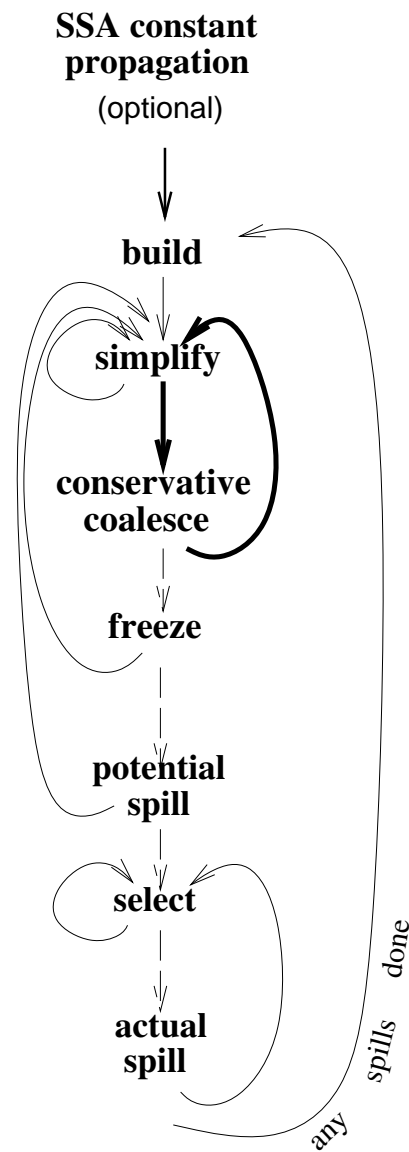
Interleave simplification with coalescing to eliminate most moves while guaranteeing not to introduce spills:

1. *Build* interference graph G and distinguish move-related from non-move-related nodes
2. *Simplify*: remove non-move-related nodes of low degree one at a time
3. *Coalesce*: conservatively coalesce move-related nodes
 - remove associated move instruction
 - if resulting node is non-move-related it can now be simplified
 - repeat simplify and coalesce until only significant-degree or uncoalesced moves
4. *Freeze*: if unable to simplify or coalesce
 - (a) look for move-related node of low-degree
 - (b) freeze its associated moves (give up on coalescing)
 - (c) now treat as non-move-related; resume iteration of simplify and coalesce

Iterated register coalescing (cont.)

5. *Spill*: if no low-degree nodes
 - (a) select candidate for spilling
 - (b) remove to stack and continue simplifying
6. *Select*: pop stack assigning colors (including actual spills)
7. *Start over*: if select has no actual spills then finished, otherwise
 - (a) rewrite code to fetch actual spills before each use and store after each definition
 - (b) recalculate liveness and repeat

Iterated register coalescing



Spilling

- Spills require repeating *build* and *simplify* on the whole program
- To avoid increasing number of spills in future rounds of *build* can simply discard coalescences
- Alternatively, preserve coalescences from before first *potential* spill, discard those after that point
- Move-related spilled temporaries can be aggressively coalesced, since (unlike registers) there is no limit on the number of stack-frame locations

Precolored nodes

Precolored nodes correspond to machine registers (e.g., stack pointer, arguments, return address, return value)

- *select* and *coalesce* can give an ordinary temporary the same color as a precolored register, if they don't interfere
- e.g., argument registers can be reused inside procedures for a temporary
- *simplify*, *freeze* and *spill* cannot be performed on them
- also, precolored nodes interfere with other precolored nodes

So, treat precolored nodes as having infinite degree

This also avoids needing to store large adjacency lists for precolored nodes; coalescing can use the George criterion

Temporary copies of machine registers

Since precolored nodes don't spill, their live ranges must be kept short:

1. use *move* instructions
2. move callee-save registers to fresh temporaries on procedure entry, and back on exit, spilling between as necessary
3. *register pressure* will spill the fresh temporaries as necessary, otherwise they can be coalesced with their precolored counterpart and the moves deleted

Caller-save and callee-save registers

Variables whose live ranges span calls should go to callee-save registers, otherwise to caller-save

This is easy for graph coloring allocation with spilling

- calls interfere with caller-save registers
- a cross-call variable interferes with all precolored caller-save registers, as well as with the fresh temporaries created for callee-save copies, forcing a spill
- choose nodes with high degree but few uses, to spill the fresh callee-save temporary instead of the cross-call variable
- this makes the original callee-save register available for coloring the cross-call variable

Example

enter:

c := r3

a := r1

b := r2

d := 0

e := a

loop:

d := d + b

e := e - 1

if e > 0 goto loop

r1 := d

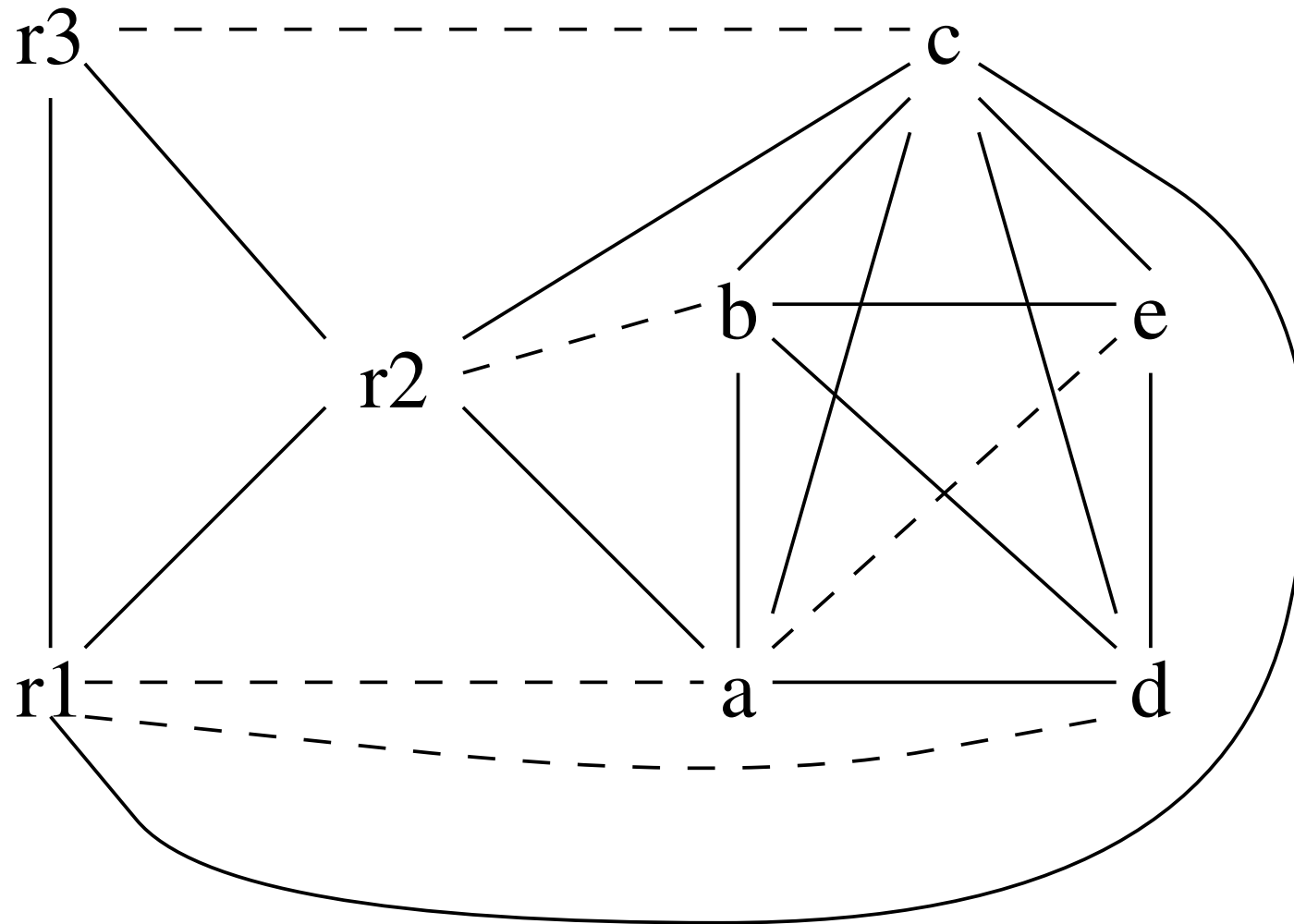
r3 := c

return [r1, r3 live out]

- Temporaries are a, b, c, d, e
- Assume target machine with $K = 3$ registers: r1, r2 (caller-save/argument/result), r3 (callee-save)
- The code generator has already made arrangements to save r3 explicitly by copying into temporary a and back again

Example (cont.)

Interference graph:



Example (cont.)

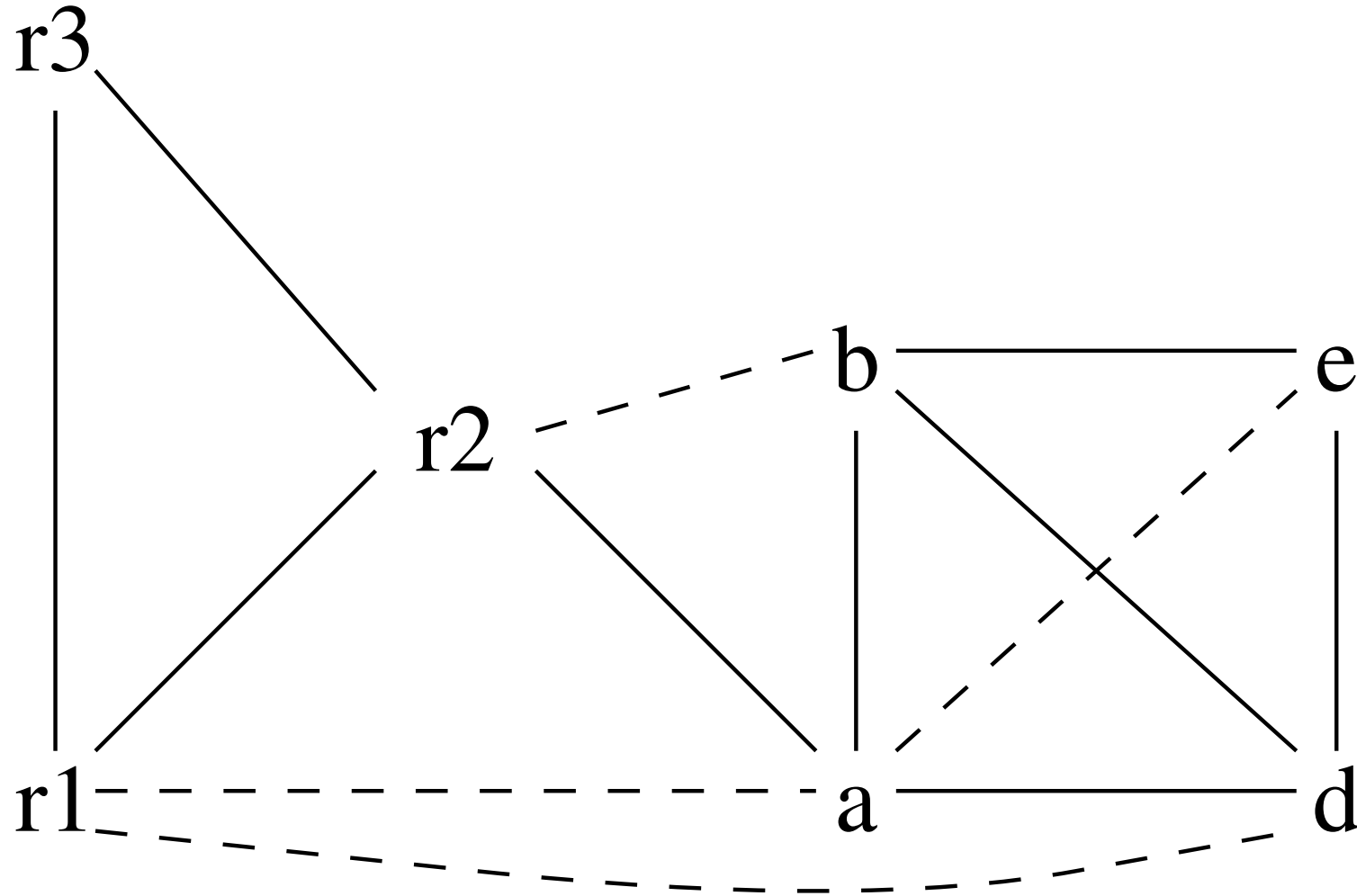
- No opportunity for *simplify* or *freeze* (all non-precolored nodes have significant degree $\geq K$)
- Any *coalesce* will produce a new node adjacent to $\geq K$ significant-degree nodes
- Must *spill* based on priorities:

Node	uses + defs outside loop		uses + defs inside loop		degree	priority
a	(2	+10×	0	4	= 0.50
b	(1	+10×	1	4	= 2.75
c	(2	+10×	0	6	= 0.33
d	(2	+10×	2	4	= 5.50
e	(1	+10×	3	3	= 10.30

- Node c has lowest priority so spill it

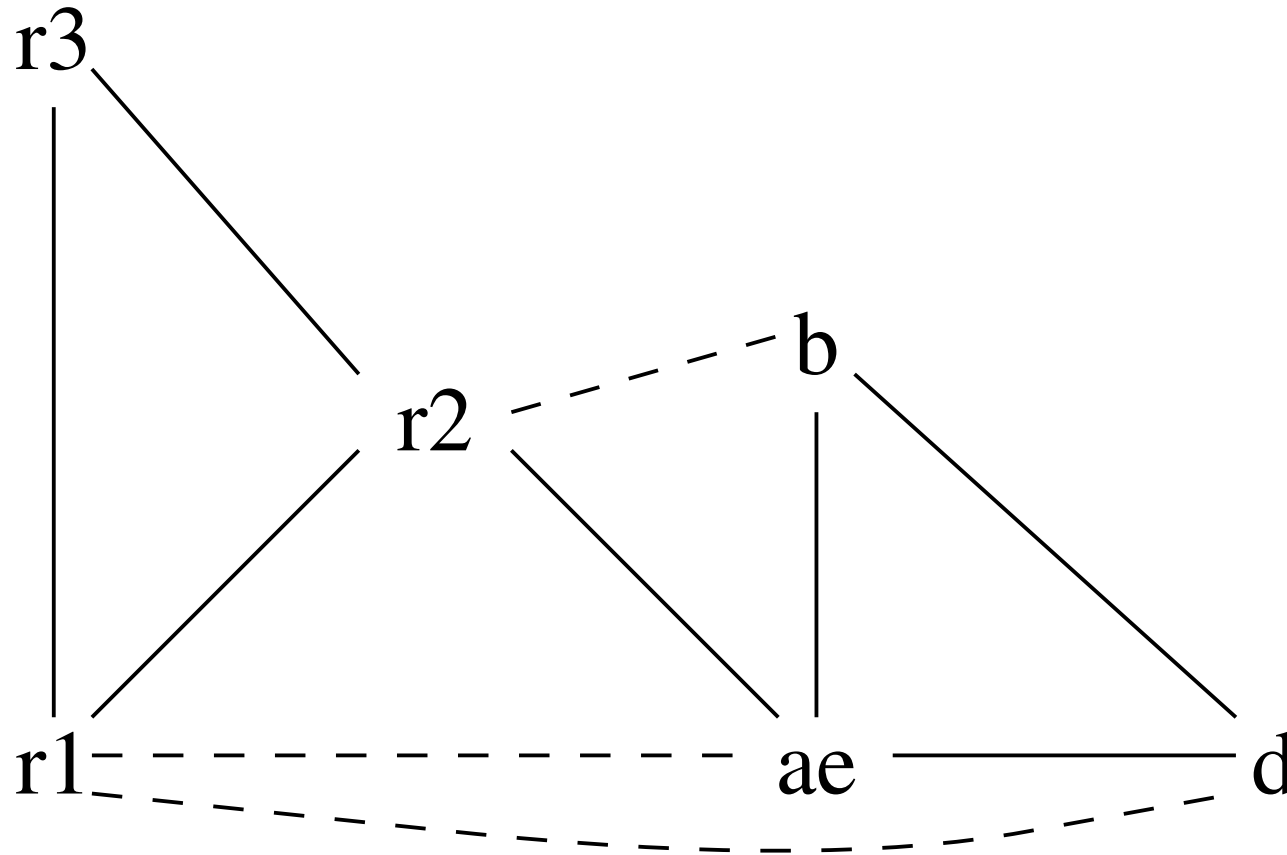
Example (cont.)

Interference graph with c removed:



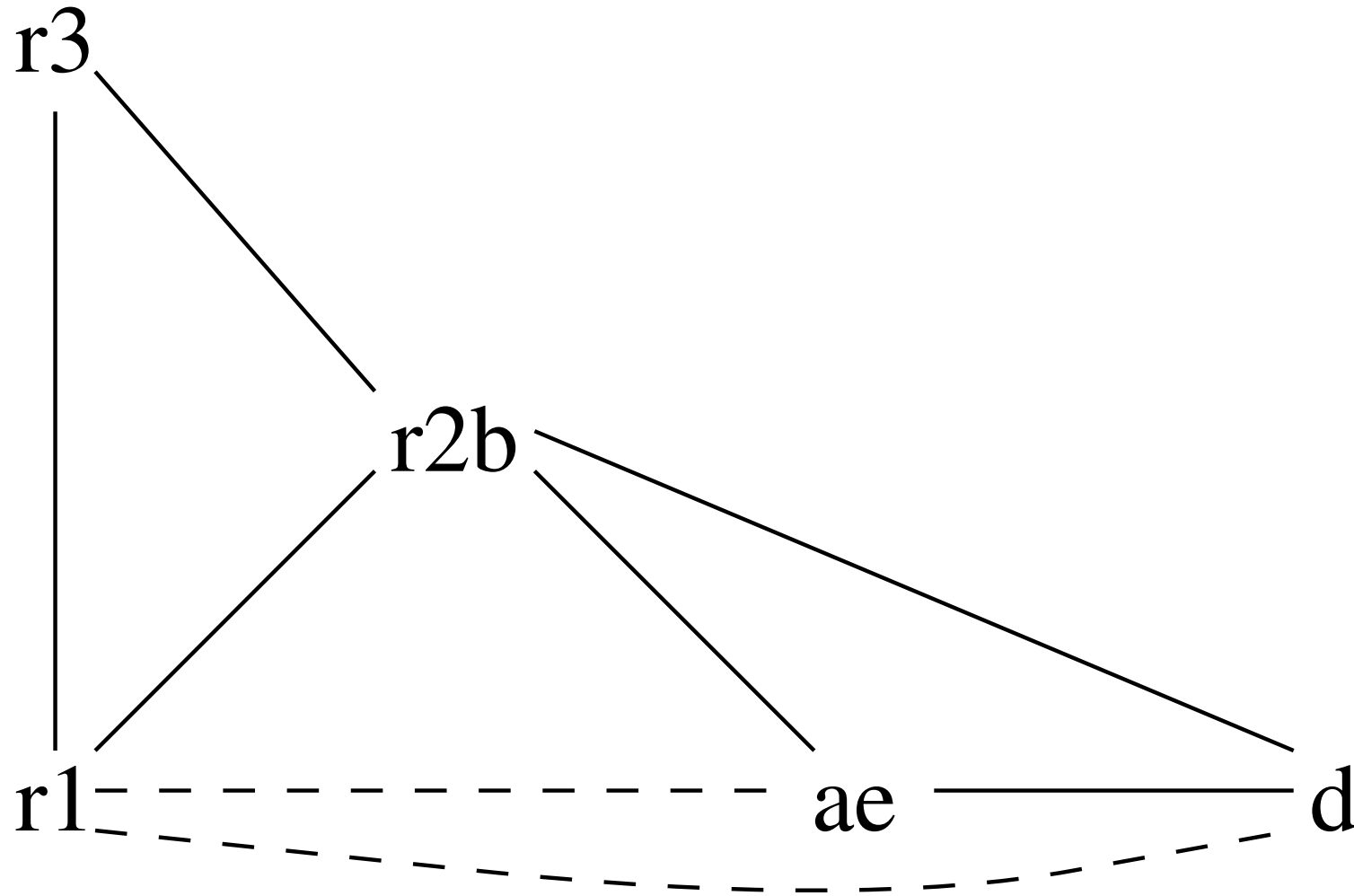
Example (cont.)

Only possibility is to *coalesce* a and e: ae will have $< K$ significant-degree neighbors (after coalescing d will be low-degree, though high-degree before)



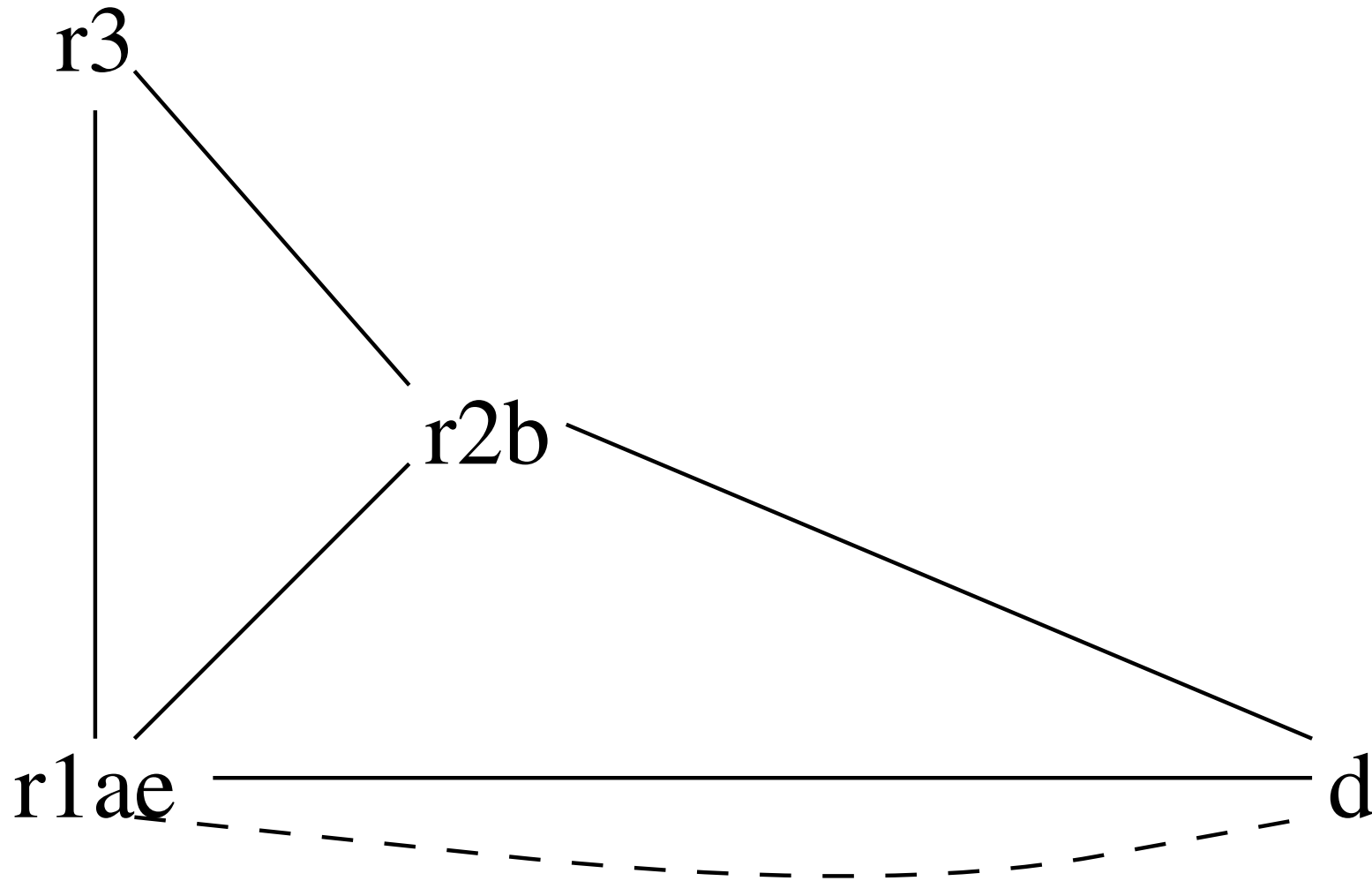
Example (cont.)

Can now *coalesce* b with r2 (or coalesce ae and r1):



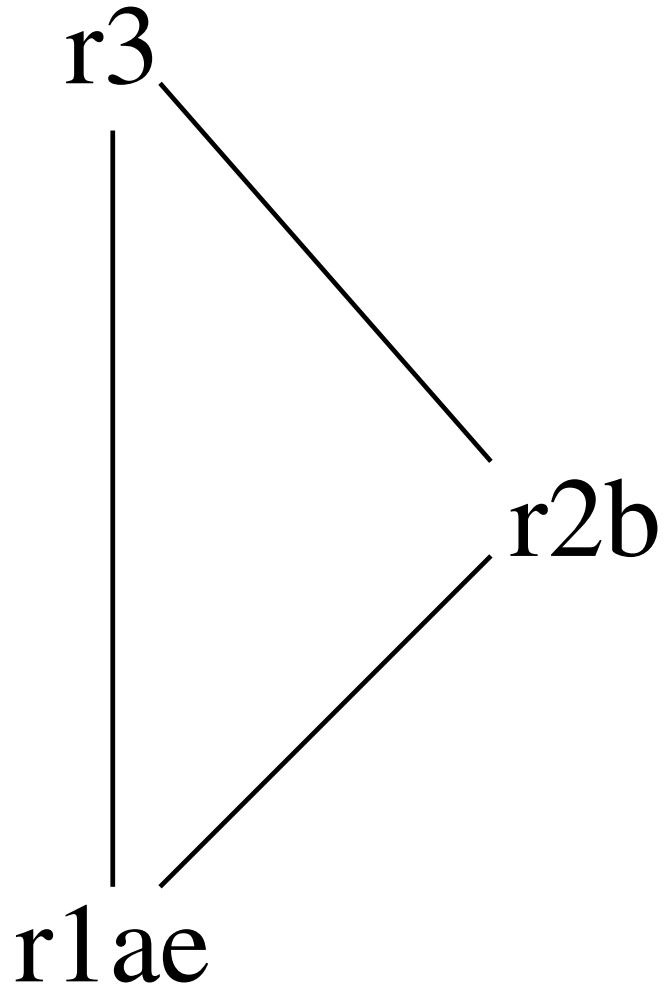
Example (cont.)

Coalescing ae and r1 (could also coalesce d with r1):



Example (cont.)

Cannot *coalesce* r1ae with d because the move is *constrained*: the nodes interfere. Must *simplify* d:



Example (cont.)

- Graph now has only precolored nodes, so pop nodes from stack coloring along the way
 - $d \equiv r3$
 - a, b, e have colors by coalescing
 - c must spill since no color can be found for it
- Introduce new temporaries c1 and c2 for each use/def, add loads before each use and stores after each def

Example (cont.)

enter:

c1 := r3

M[c_loc] := c1

a := r1

b := r2

d := 0

e := a

loop:

d := d + b

e := e - 1

if e > 0 goto loop

r1 := d

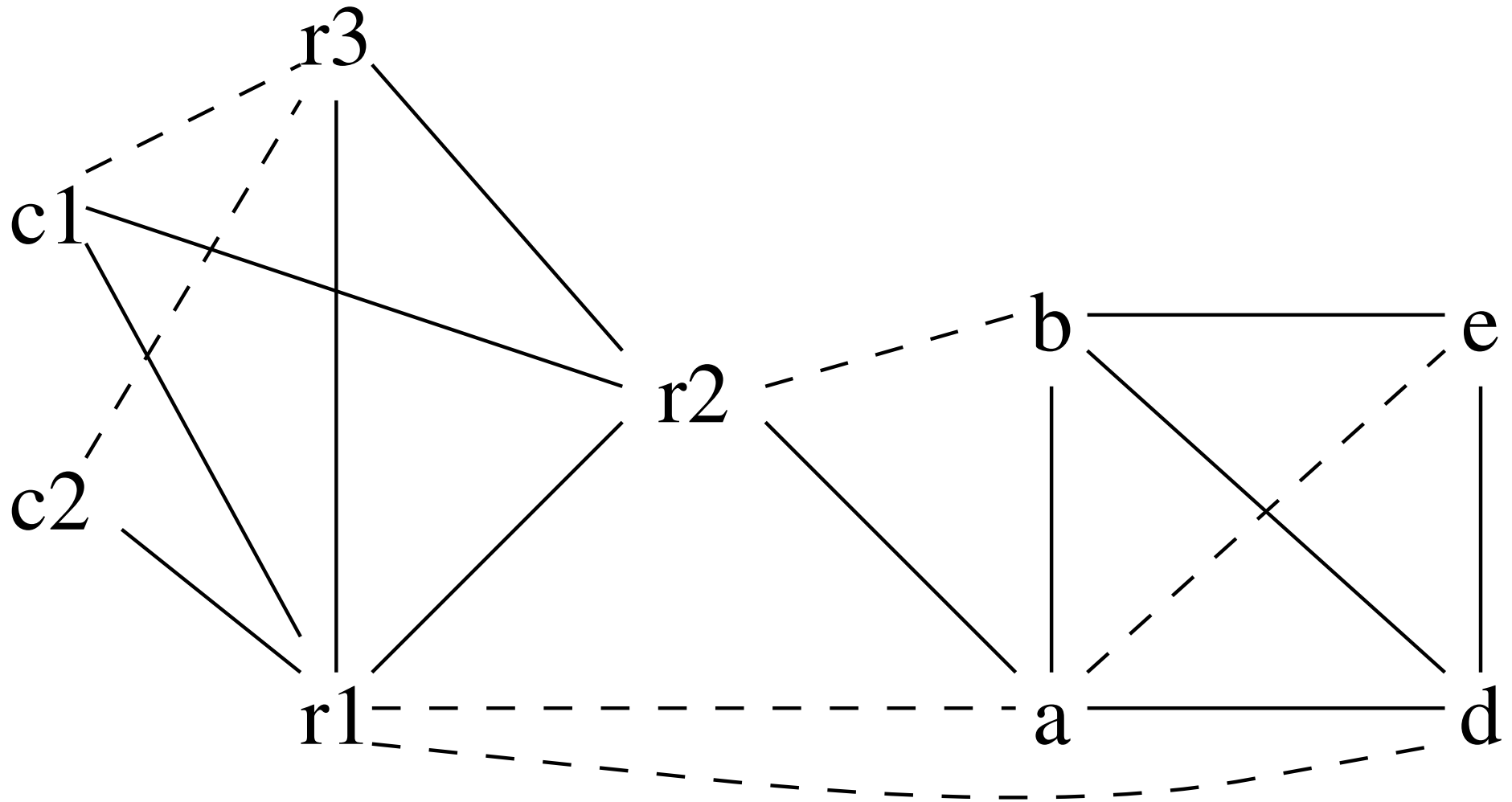
c2 := M[c_loc]

r3 := c2

return [r1, r3 live out]

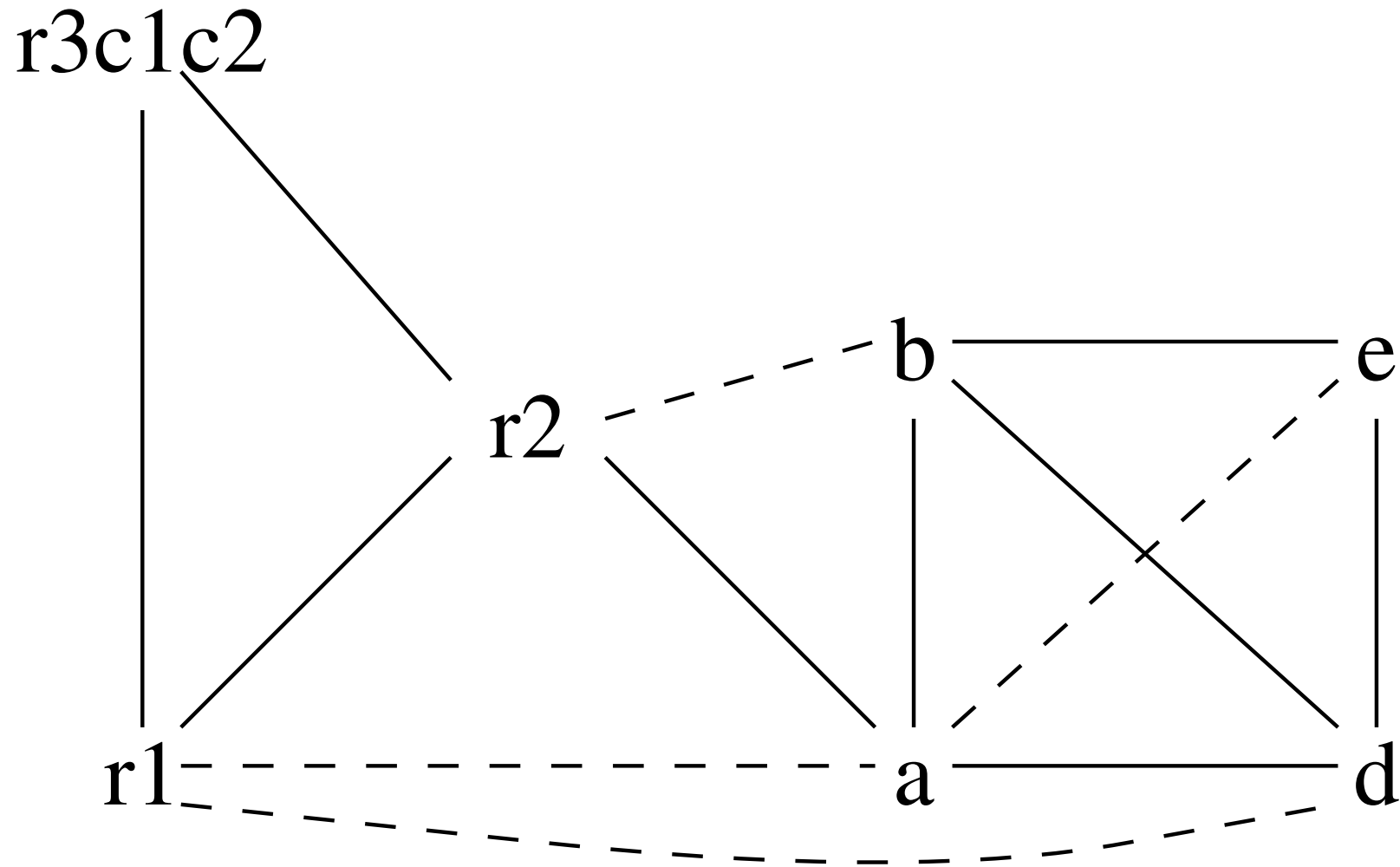
Example (cont.)

New interference graph:



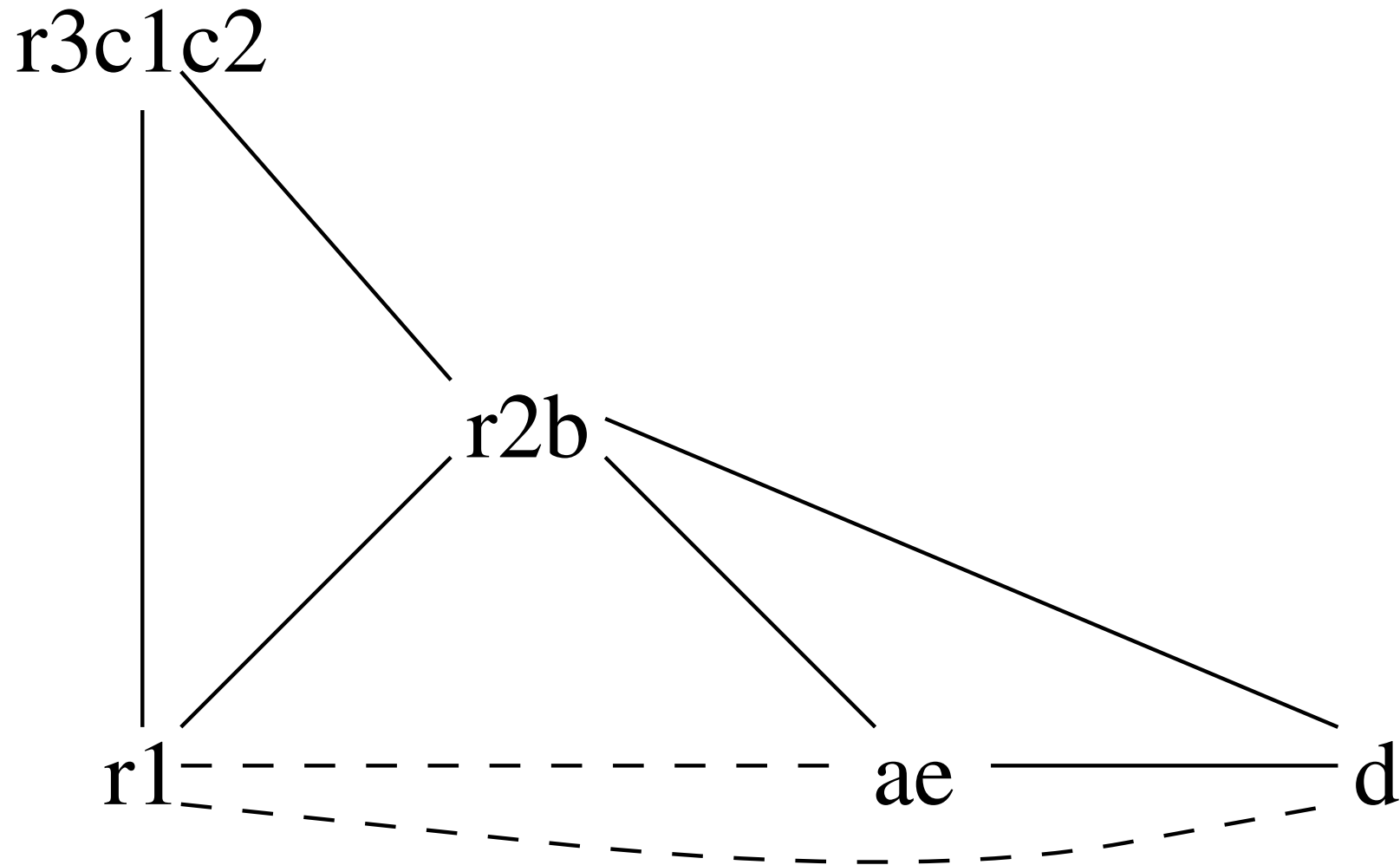
Example (cont.)

Coalesce c1 with r3, then c2 with r3:



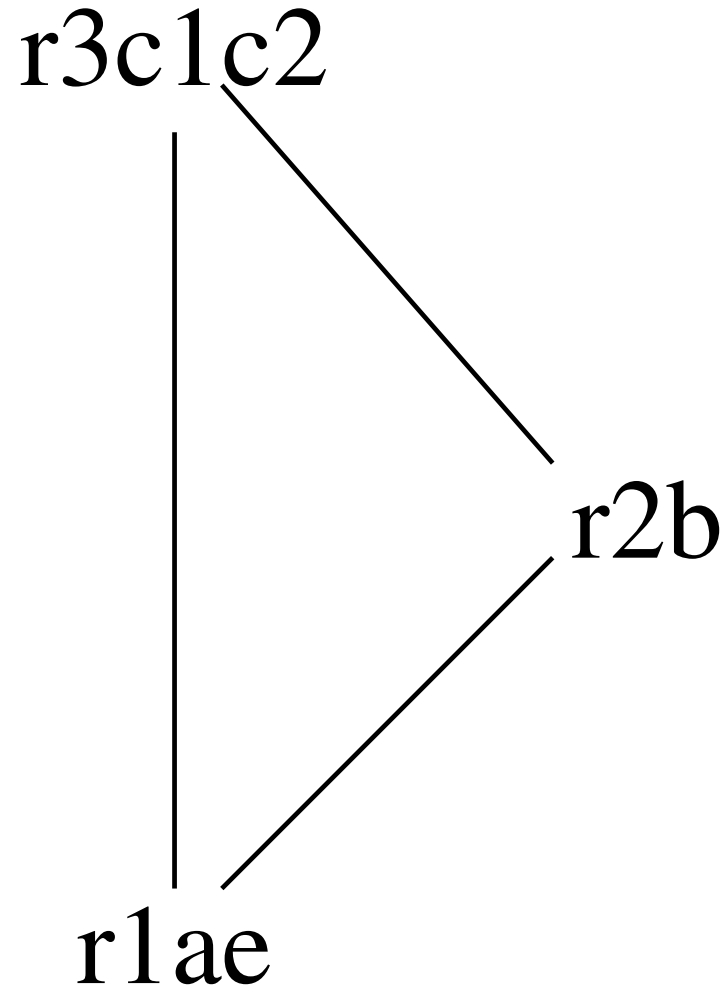
Example (cont.)

As before, *coalesce* a with e, then b with r2:



Example (cont.)

As before, *coalesce* ae with r1 and *simplify* d:



Example (cont.)

Pop d from stack: select $r3$. All other nodes were coalesced or precolored. So, the coloring is:

- $a \equiv r1$
- $b \equiv r2$
- $c \equiv r3$
- $d \equiv r3$
- $e \equiv r1$

Example (cont.)

Rewrite the program with this assignment:

enter:

 r3 := r3

 M[c_loc] := r3

 r1 := r1

 r2 := r2

 r3 := 0

 r1 := r1

loop:

 r3 := r3 + r2

 r1 := r1 - 1

 if r1 > 0 goto loop

 r1 := r3

 r3 := M[c_loc]

 r3 := r3

 return [r1, r3 live out]

Example (cont.)

- Delete moves with source and destination the same (coalesced):

enter:

 M[c_loc] := r3

 r3 := 0

loop:

 r2 := r3 + r2

 r1 := r1 - 1

 if r1 > 0 goto loop

 r1 := r3

 r3 := M[c_loc]

 return [r1, r3 live out]

- One uncoalesced move remains