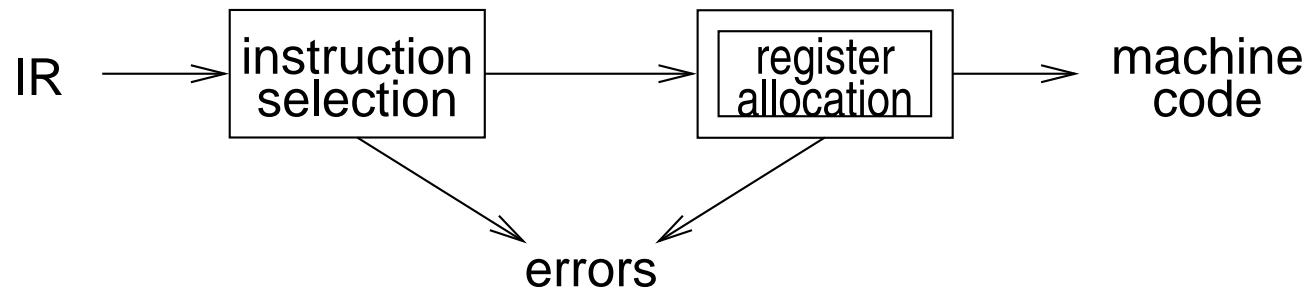


Register Allocation

Copyright ©2023 by Antony L. Hosking. *Permission to make digital or hard copies of part or all of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and full citation on the first page. To copy otherwise, to republish, to post on servers, or to redistribute to lists, requires prior specific permission and/or fee. Request permission to publish from hosking@acm.org.*

Register allocation



Register allocation:

- have value in a register when used
- limited resources
- changes instruction choices
- can move loads and stores
- optimal allocation is difficult
 \Rightarrow NP-complete for $k \geq 1$ registers

Register allocation by simplification

Assume K registers

1. *Build* interference graph G : for each program point
 - (a) compute set of temporaries simultaneously live
 - (b) add edge to graph for each pair in set
2. *Simplify*: Color graph using a simple heuristic
 - (a) suppose G has node m with degree $< K$
 - (b) if $G' = G - \{m\}$ can be colored then so can G , since nodes adjacent to m have at most $K - 1$ colors
 - (c) each such simplification will reduce degree of remaining nodes leading to more opportunity for simplification
 - (d) leads to recursive coloring algorithm
3. *Spill*: suppose $\exists m$ of degree $< K$
 - (a) target some node (temporary) for spilling (optimistically, spilling node will allow coloring of remaining nodes)
 - (b) remove and continue simplifying

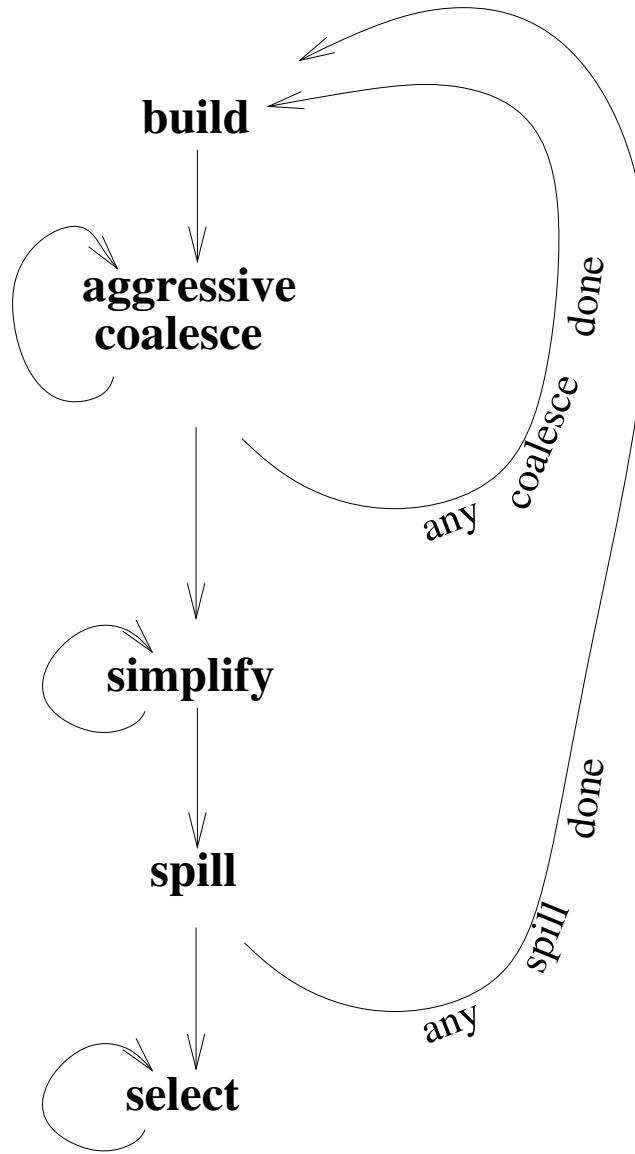
Register allocation by simplification (cont.)

4. *Select*: assign colors to nodes
 - (a) start with empty graph
 - (b) must be a color for non-spill nodes (basis for removal)
 - (c) if adding spill node and no color available (neighbors already K-colored) then mark as an *actual spill*
 - (d) repeat select
5. *Start over*: if select has no actual spills then finished, otherwise
 - (a) rewrite program to fetch actual spills before each use and store after each definition
 - (b) recalculate liveness and repeat

Coalescing

- Can delete a *move* instruction when source s and destination d do not interfere:
 - *coalesce* them into a new node whose edges are the union of those of s and d
- In principle, any pair of non-interfering nodes can be coalesced
 - unfortunately, the union is more constrained and new graph may no longer be K -colorable
 - overly aggressive

Simplification with aggressive coalescing



Conservative coalescing

Apply tests for coalescing that preserve colorability.

Suppose a and b are candidates for coalescing into node ab .

Briggs: coalesce only if ab has $< K$ neighbors of *significant* degree $\geq K$

- *simplify* first removes all insignificant-degree neighbors
- ab will then be adjacent to $< K$ neighbors
- *simplify* can then remove ab

George: coalesce only if all significant-degree neighbors of a already interfere with b

- *simplify* removes all insignificant-degree neighbors of a
- remaining significant-degree neighbors of a already interfere with b so coalescing does not increase the degree of any node

Iterated register coalescing

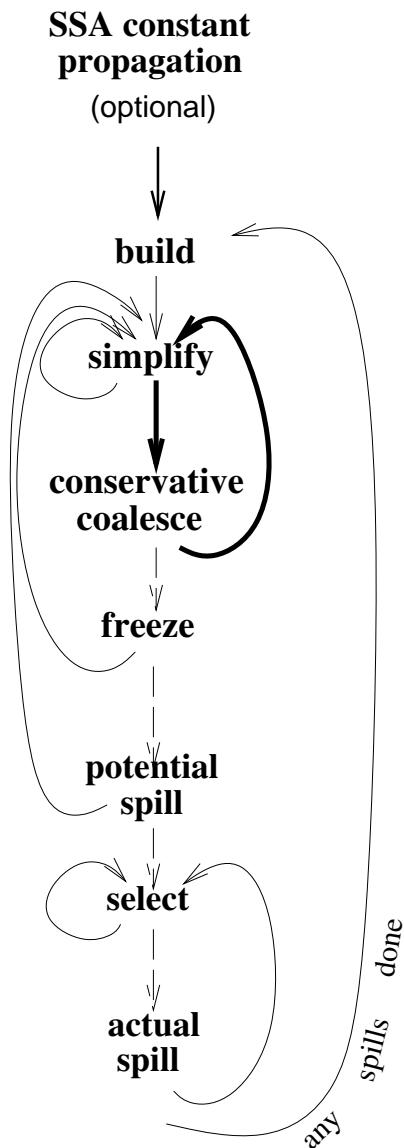
Interleave simplification with coalescing to eliminate most moves while guaranteeing not to introduce spills:

1. *Build* interference graph G and distinguish move-related from non-move-related nodes
2. *Simplify*: remove non-move-related nodes of low degree one at a time
3. *Coalesce*: conservatively coalesce move-related nodes
 - remove associated move instruction
 - if resulting node is non-move-related it can now be simplified
 - repeat simplify and coalesce until only significant-degree or uncoalesced moves
4. *Freeze*: if unable to simplify or coalesce
 - (a) look for move-related node of low-degree
 - (b) freeze its associated moves (give up on coalescing)
 - (c) now treat as non-move-related; resume iteration of simplify and coalesce

Iterated register coalescing (cont.)

5. *Spill*: if no low-degree nodes
 - (a) select candidate for spilling
 - (b) remove to stack and continue simplifying
6. *Select*: pop stack assigning colors (including actual spills)
7. *Start over*: if select has no actual spills then finished, otherwise
 - (a) rewrite code to fetch actual spills before each use and store after each definition
 - (b) recalculate liveness and repeat

Iterated register coalescing



Spilling

- Spills require repeating *build* and *simplify* on the whole program
- To avoid increasing number of spills in future rounds of *build* can simply discard coalescences
- Alternatively, preserve coalescences from before first *potential* spill, discard those after that point
- Move-related spilled temporaries can be aggressively coalesced, since (unlike registers) there is no limit on the number of stack-frame locations

Precolored nodes

Precolored nodes correspond to machine registers (e.g., stack pointer, arguments, return address, return value)

- *select* and *coalesce* can give an ordinary temporary the same color as a precolored register, if they don't interfere
- e.g., argument registers can be reused inside procedures for a temporary
- *simplify*, *freeze* and *spill* cannot be performed on them
- also, precolored nodes interfere with other precolored nodes

So, treat precolored nodes as having infinite degree

This also avoids needing to store large adjacency lists for precolored nodes; coalescing can use the George criterion

Temporary copies of machine registers

Since precolored nodes don't spill, their live ranges must be kept short:

1. use *move* instructions
2. move callee-save registers to fresh temporaries on procedure entry, and back on exit, spilling between as necessary
3. *register pressure* will spill the fresh temporaries as necessary, otherwise they can be coalesced with their precolored counterpart and the moves deleted

Caller-save and callee-save registers

Variables whose live ranges span calls should go to callee-save registers, otherwise to caller-save

This is easy for graph coloring allocation with spilling

- calls interfere with caller-save registers
- a cross-call variable interferes with all precolored caller-save registers, as well as with the fresh temporaries created for callee-save copies, forcing a spill
- choose nodes with high degree but few uses, to spill the fresh callee-save temporary instead of the cross-call variable
- this makes the original callee-save register available for coloring the cross-call variable

Example

enter:

```
c := r3
a := r1
b := r2
d := 0
e := a
```

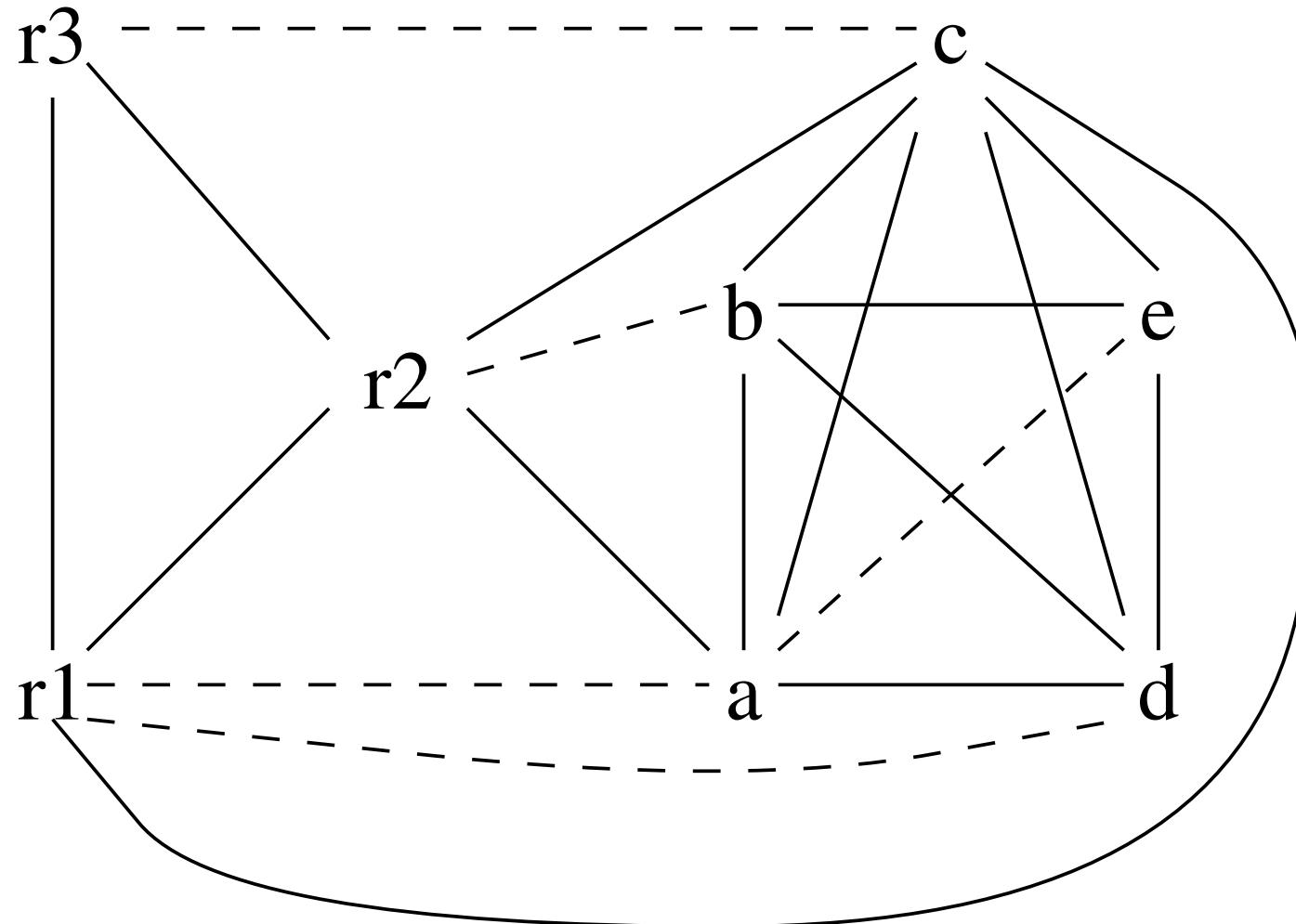
loop:

```
d := d + b
e := e - 1
if e > 0 goto loop
r1 := d
r3 := c
return [ r1, r3 live out ]
```

- Temporaries are a, b, c, d, e
- Assume target machine with $K = 3$ registers: r1, r2 (caller-save/argument/result), r3 (callee-save)
- The code generator has already made arrangements to save r3 explicitly by copying into temporary a and back again

Example (cont.)

Interference graph:



Example (cont.)

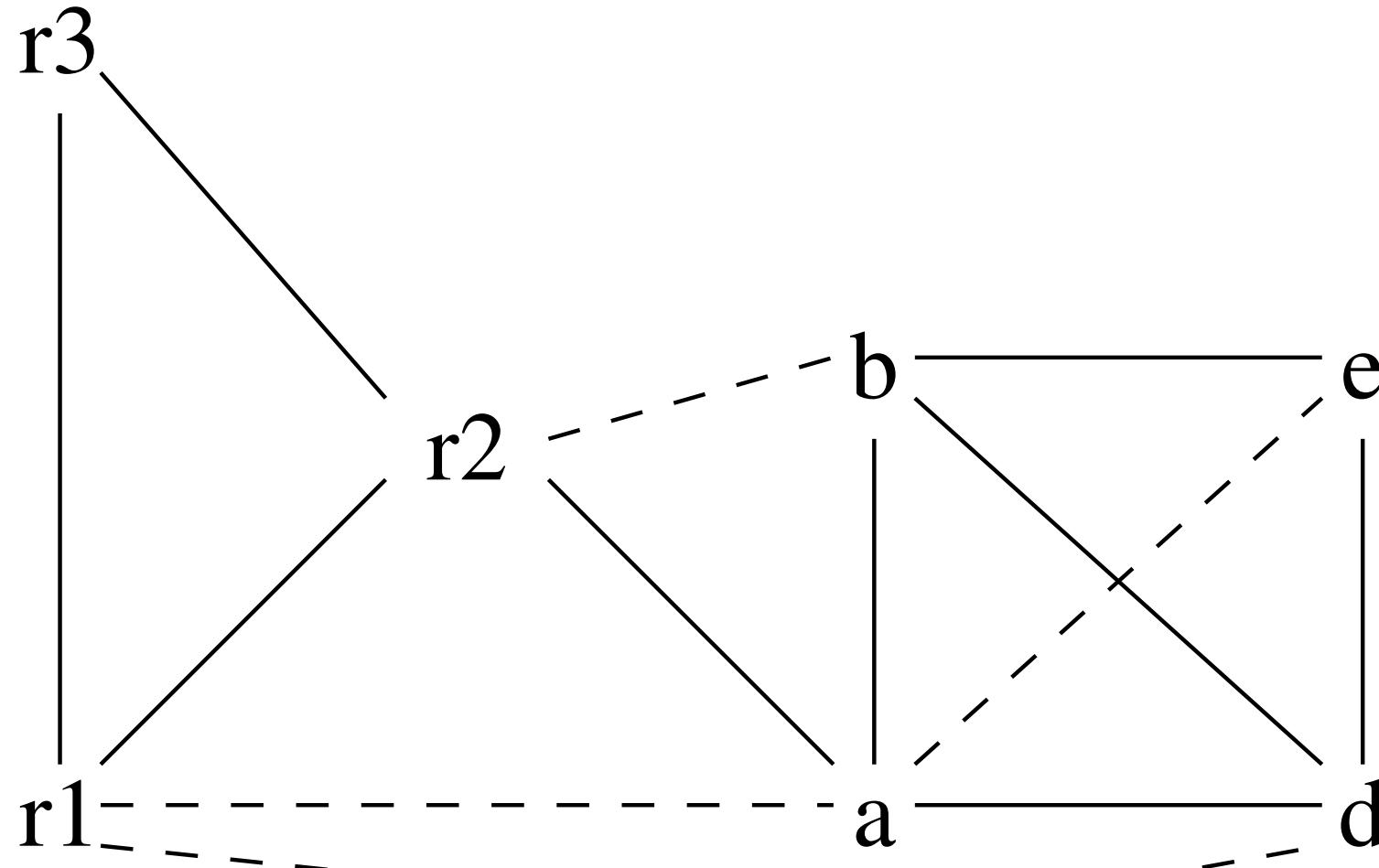
- No opportunity for *simplify* or *freeze* (all non-precolored nodes have significant degree $\geq K$)
- Any *coalesce* will produce a new node adjacent to $\geq K$ significant-degree nodes
- Must *spill* based on priorities:

Node	uses + defs outside loop	uses + defs inside loop	degree	priority
a	2	0	4	0.50
b	1	1	4	2.75
c	2	0	6	0.33
d	2	2	4	5.50
e	1	3	3	10.30

- Node c has lowest priority so spill it

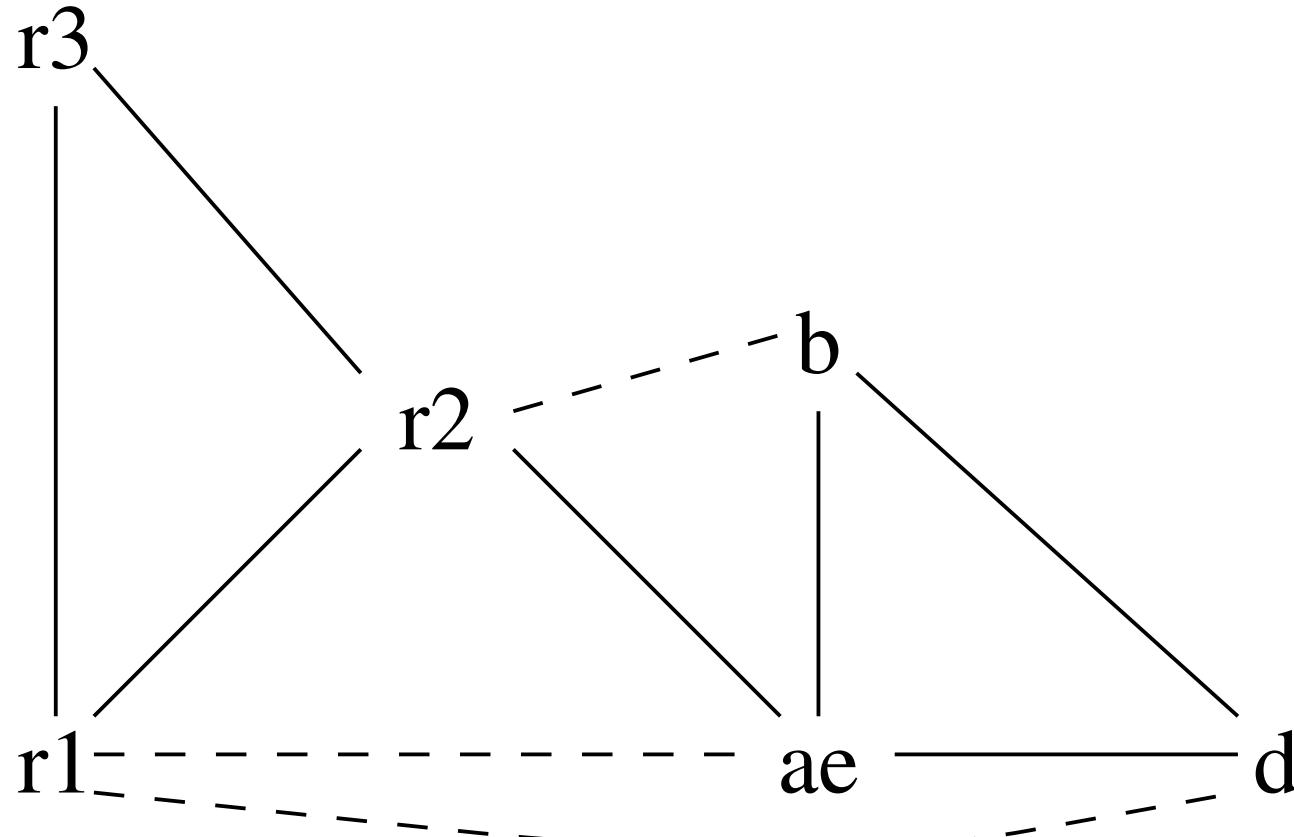
Example (cont.)

Interference graph with c removed:



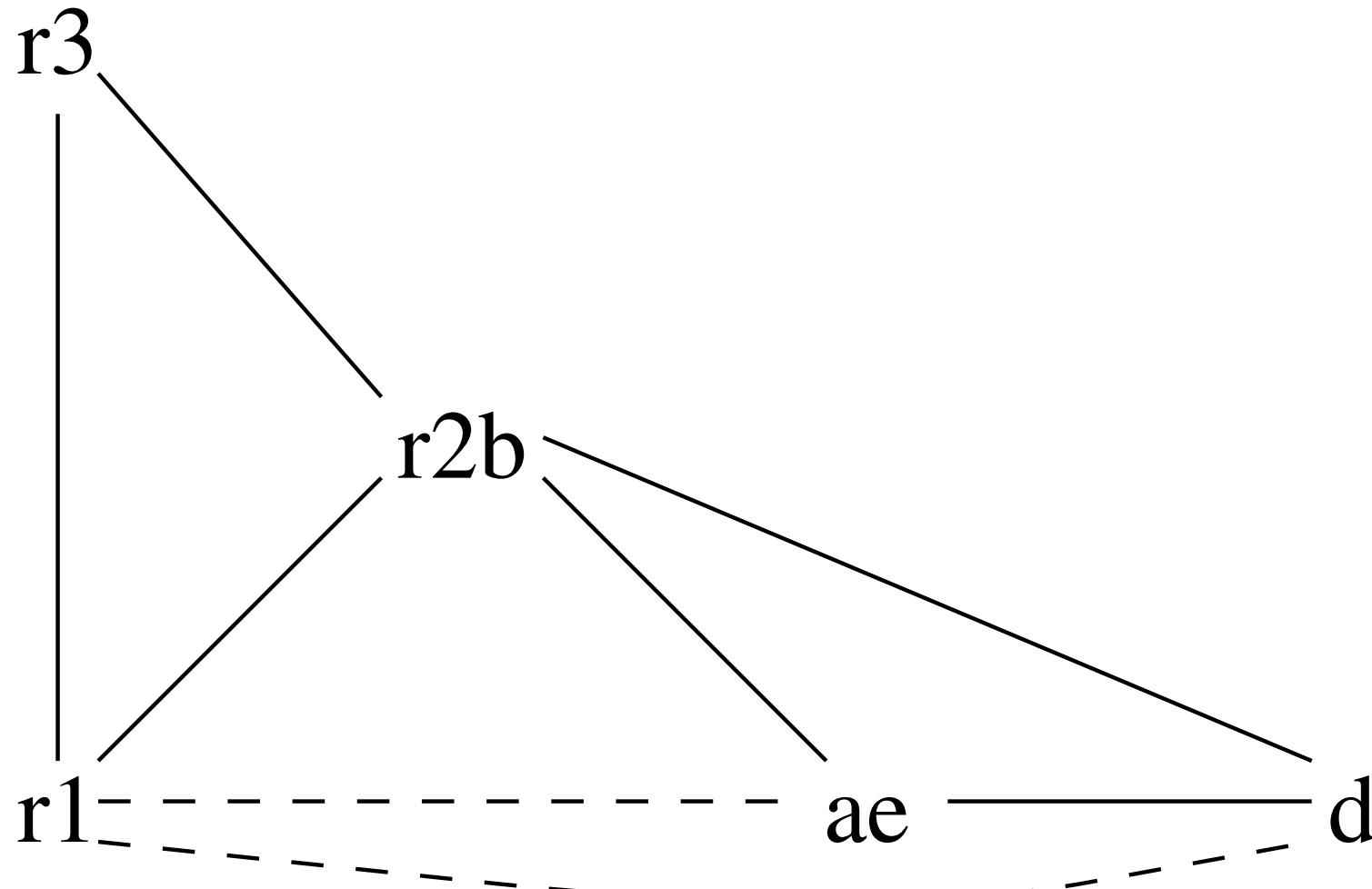
Example (cont.)

Only possibility is to *coalesce* a and e: ae will have $< K$ significant-degree neighbors (after coalescing d will be low-degree, though high-degree before)



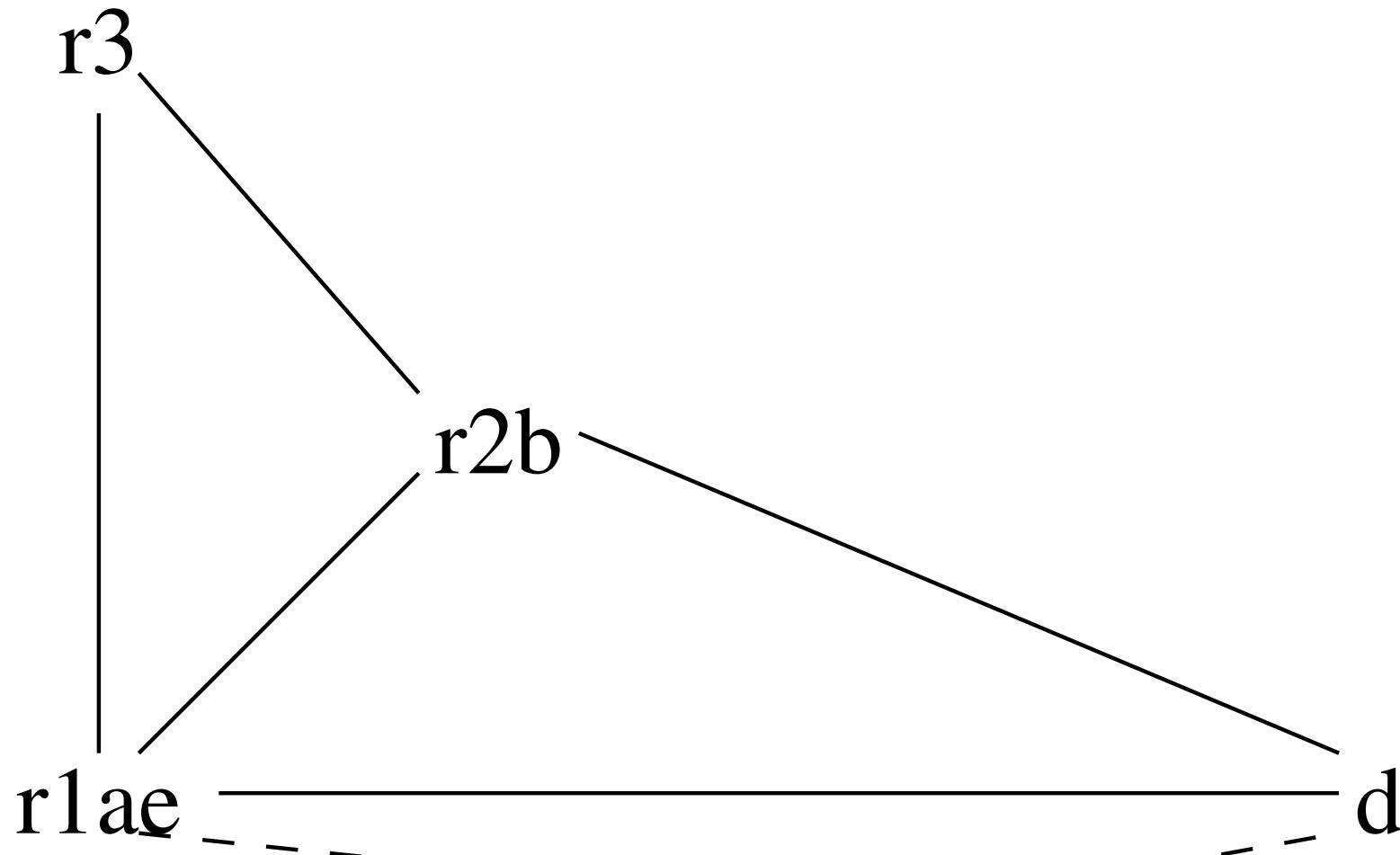
Example (cont.)

Can now *coalesce* b with r2 (or coalesce ae and r1):



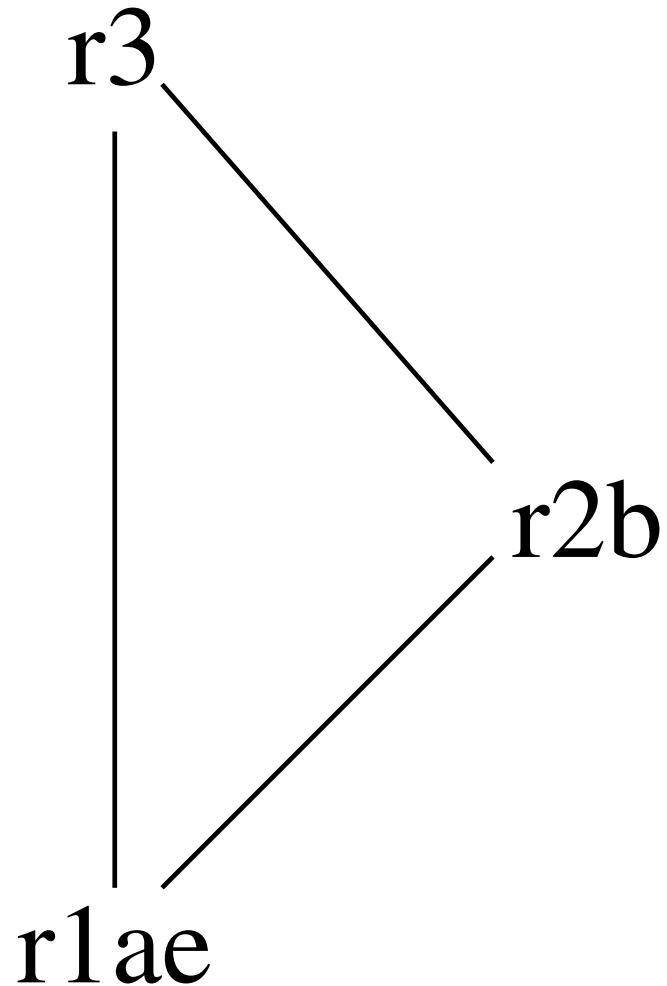
Example (cont.)

Coalescing r_3 and r_1 (could also coalesce d with r_1):



Example (cont.)

Cannot *coalesce* r_{1ae} with d because the move is *constrained*: the nodes interfere. Must *simplify* d :



Example (cont.)

- Graph now has only precolored nodes, so pop nodes from stack coloring along the way
 - $d \equiv r3$
 - a, b, e have colors by coalescing
 - c must spill since no color can be found for it
- Introduce new temporaries $c1$ and $c2$ for each use/def, add loads before each use and stores after each def

Example (cont.)

enter:

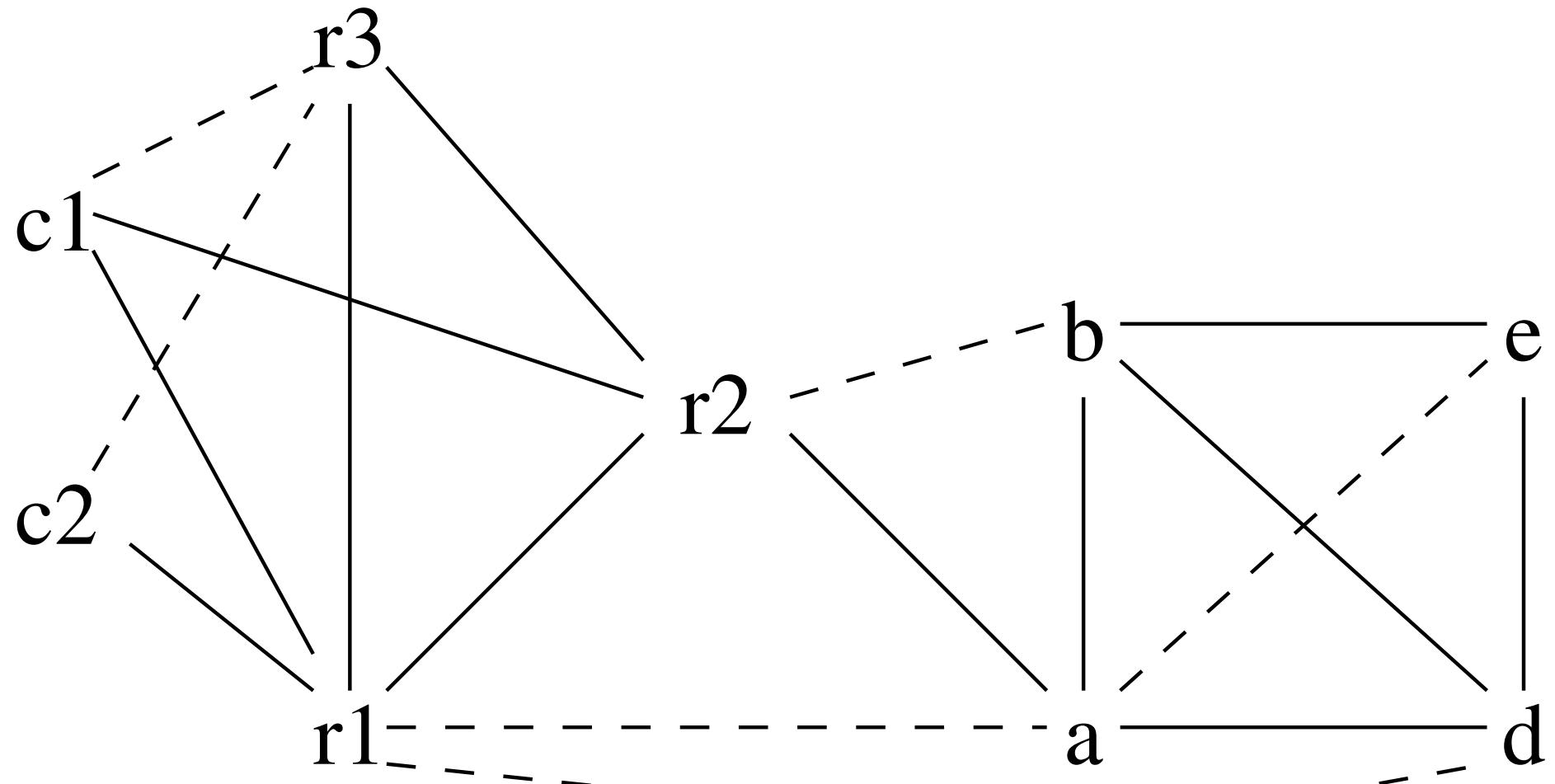
```
c1 := r3
M[c_loc] := c1
a := r1
b := r2
d := 0
e := a
```

loop:

```
d := d + b
e := e - 1
if e > 0 goto loop
r1 := d
c2 := M[c_loc]
r3 := c2
return [ r1, r3 live out ]
```

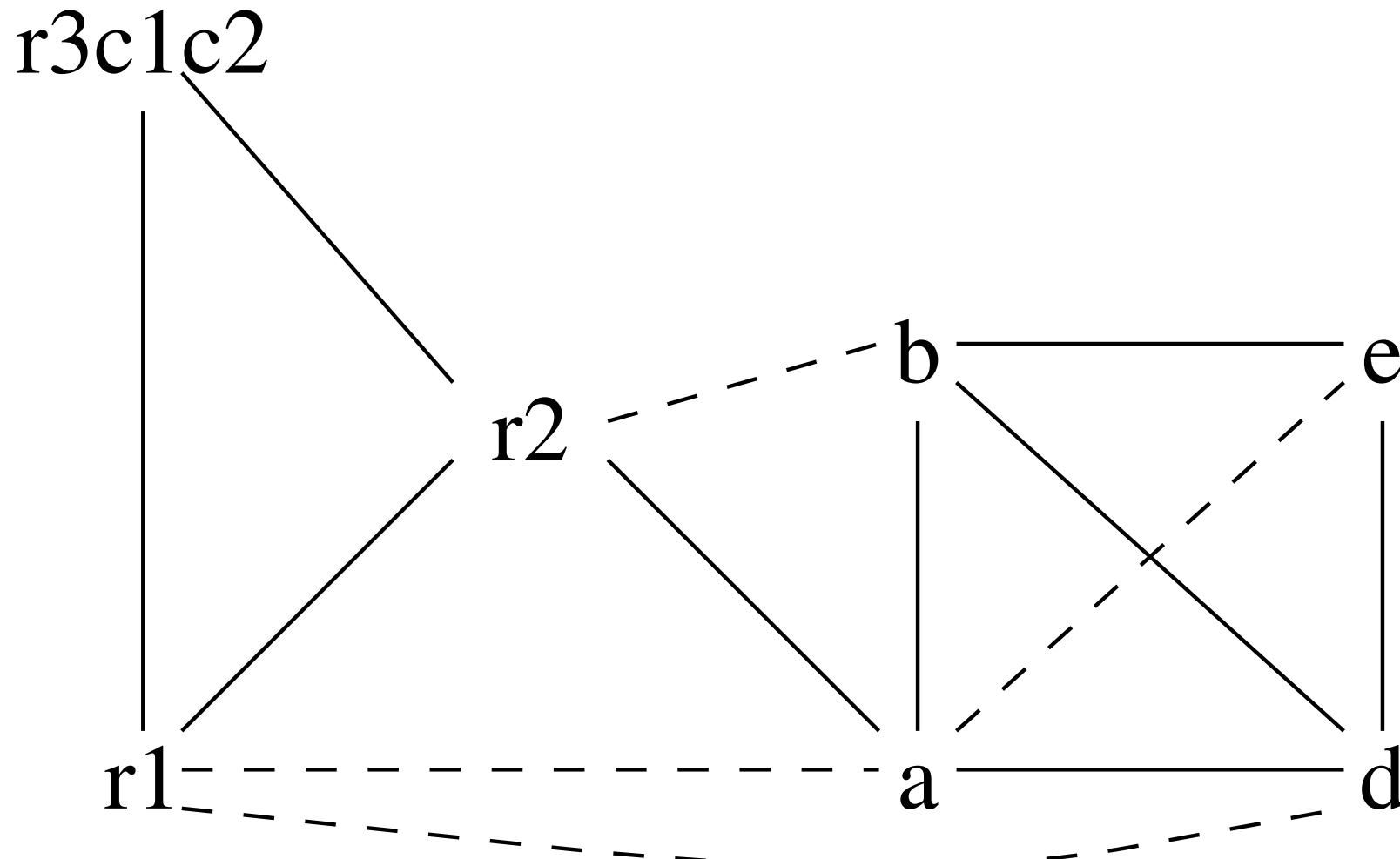
Example (cont.)

New interference graph:



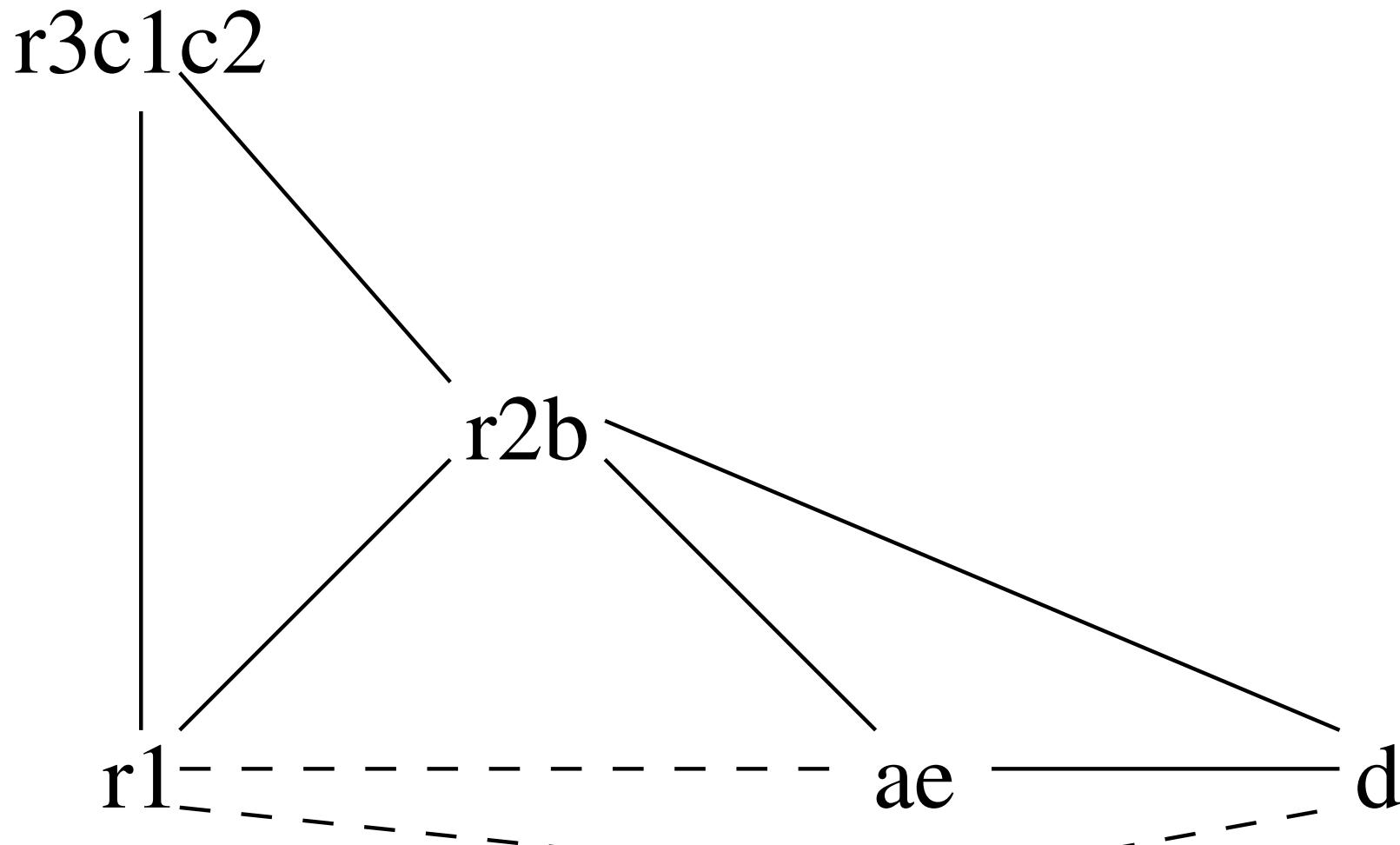
Example (cont.)

Coalesce c_1 with r_3 , then c_2 with r_3 :



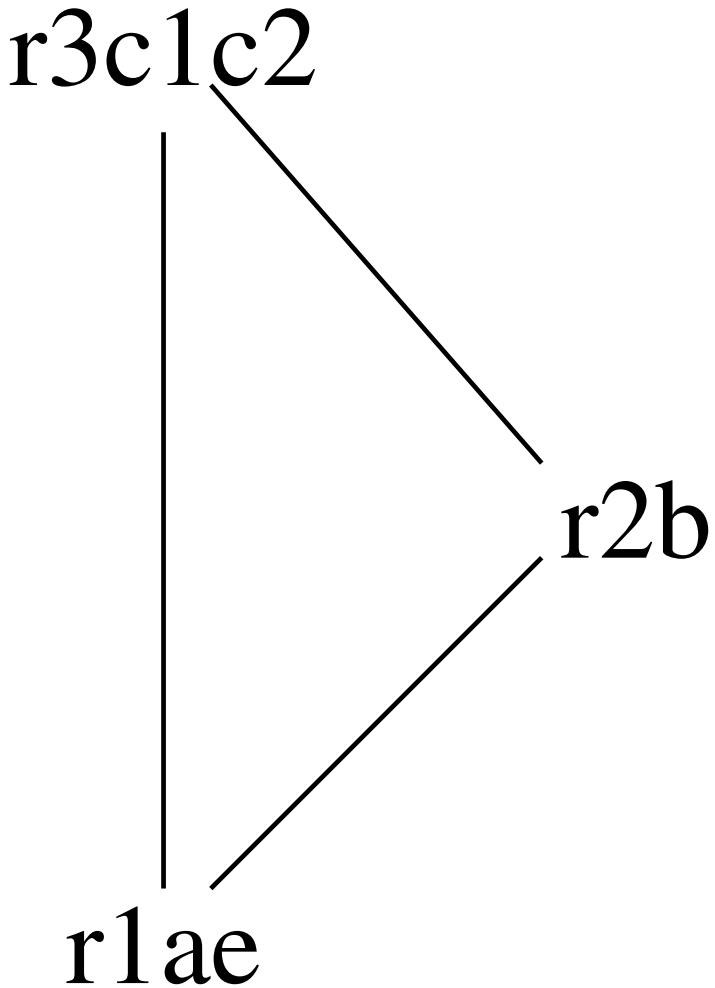
Example (cont.)

As before, *coalesce* a with e, then b with r2:



Example (cont.)

As before, *coalesce* *ae* with *r1* and *simplify d*:



Example (cont.)

Pop d from stack: select r3. All other nodes were coalesced or precolored. So, the coloring is:

- $a \equiv r1$
- $b \equiv r2$
- $c \equiv r3$
- $d \equiv r3$
- $e \equiv r1$

Example (cont.)

Rewrite the program with this assignment:

enter:

```
r3 := r3
M[c_loc] := r3
r1 := r1
r2 := r2
r3 := 0
r1 := r1
```

loop:

```
r3 := r3 + r2
r1 := r1 - 1
if r1 > 0 goto loop
r1 := r3
r3 := M[c_loc]
r3 := r3
return [ r1, r3 live out ]
```

Example (cont.)

- Delete moves with source and destination the same (coalesced):

enter:

```
M[c_loc] := r3
```

```
r3 := 0
```

loop:

```
r2 := r3 + r2
```

```
r1 := r1 - 1
```

```
if r1 > 0 goto loop
```

```
r1 := r3
```

```
r3 := M[c_loc]
```

```
return [ r1, r3 live out ]
```

- One uncoalesced move remains