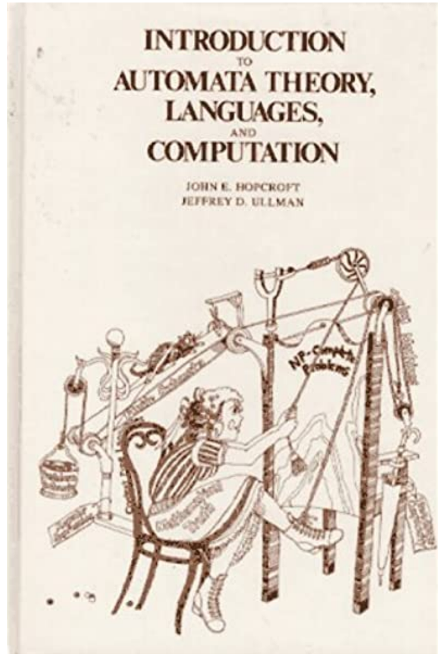
CakeML: Verified Computation/ Compilation Stories

\+ 

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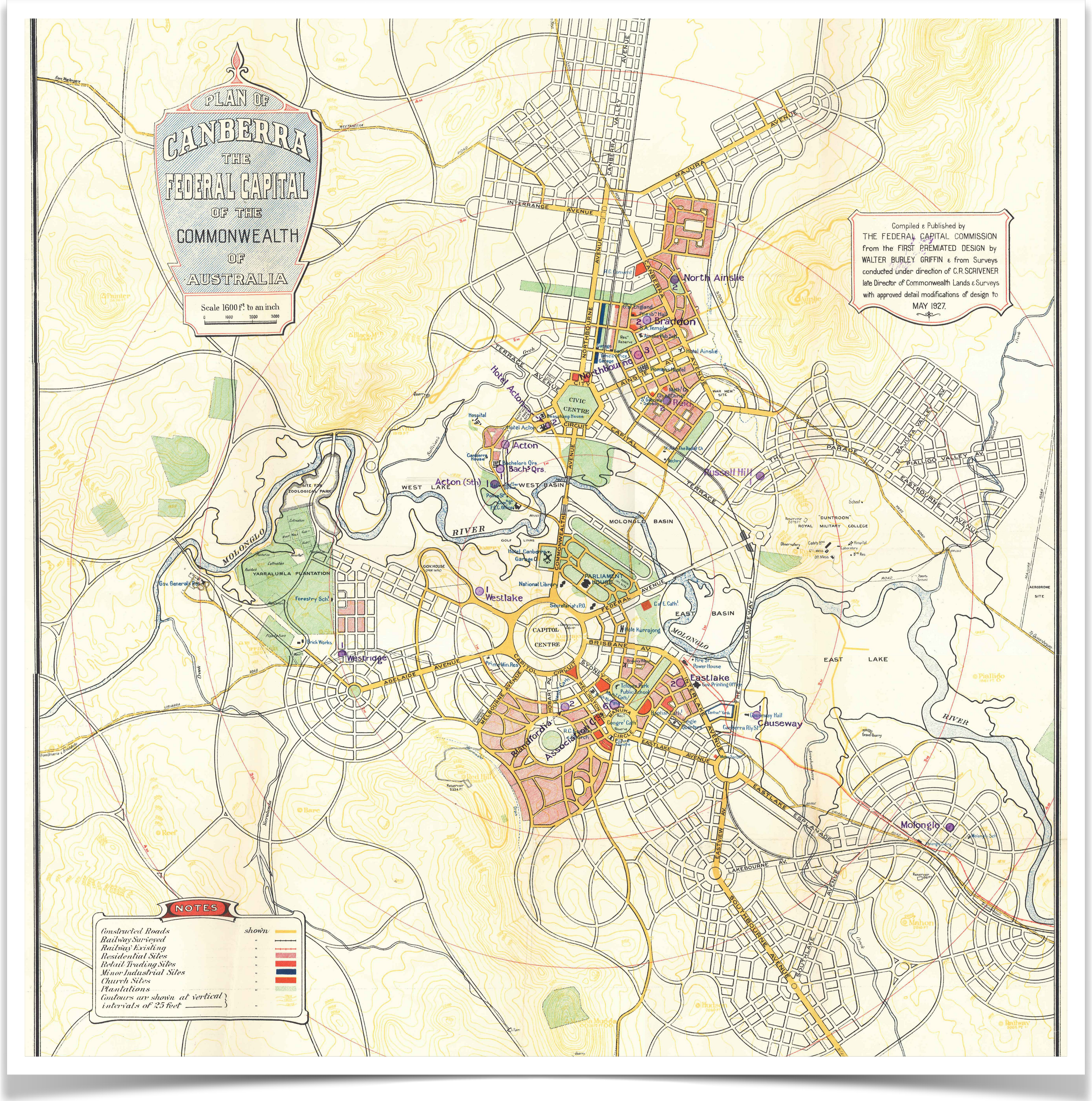
Michael Norrish, School of Computing, ANU
23 May 2023

Itinerary



Canberra Plan, 1927. Archives of the ACT Government *via* flickr.com

- Interactive theorem-proving
- CakeML
- Rendering super-standard formal language theory with maximum cleanness—puzzle included.

“Solving the world’s problems with theorem-proving.”

–me, in moments of delusion

Interactive Theorem-Proving

Interactive theorem-proving is core to 99% of what I do.

Example systems: ACL2, Coq, HOL4, HOL Light, Isabelle and PVS

Proof in Action

```
> g ‘∀n a b c. 2 < n ⇒ a ** n + b ** n ≠ c ** n’;
val it =
   Proof manager status: 1 proof.
1. Incomplete goalstack:
     Initial goal:

     ∀n a b c. 2 < n ⇒ a ** n + b ** n ≠ c ** n

:
   proofs
> e Induct;
OK..
2 subgoals:
val it =

∀a b c. 2 < SUC n ⇒ a ** SUC n + b ** SUC n ≠ c ** SUC n
------------------------------------
  ∀a b c. 2 < n ⇒ a ** n + b ** n ≠ c ** n


∀a b c. 2 < 0 ⇒ a ** 0 + b ** 0 ≠ c ** 0
```

- Human guided proof
- Posit the goal; provide the proof
- Machine helps:
  - by checking validity of steps
  - with some automatic tools
- Thus: “*proof assistant*” term

HOL4

- Under development since the mid 1980s
- Engineered in the "LCF tradition": a small kernel minimises the TCB
- Similar logic to Isabelle/HOL and HOL Light
- Simpler logic than (e.g.) Coq's.

CakeML

What?

CakeML

What?

1. A programming language in the style of Standard ML and OCaml.

CakeML

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strict evaluation, stateful

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CakeML

What?

strict evaluation, stateful

1. A programming language in the style of Standard ML and OCaml.
2. An ***ecosystem*** of proofs and verification tools
3. A ***verified, end-to-end compiler***

What Do These Words Mean? (I)

“A programming language in the style of SML and OCaml…”

(functional, has pattern-matching, nice recursion, nice datatypes)

But:

Strict: arguments are evaluated before being passed to functions. (*Unlike* Haskell.)

Stateful: CakeML supports variables that you can update by *assigning* to them. (Again, *unlike* Haskell.)

What Do These Words Mean? (II)

Verified: CakeML has proofs that guarantee it will behave correctly

End-to-end: The proofs are about "all of it":

- From: reading in the input file (a stream of characters)
- To: actual machine code for the CPU (x86, ARM etc)

Dimensions of Compiler Verification

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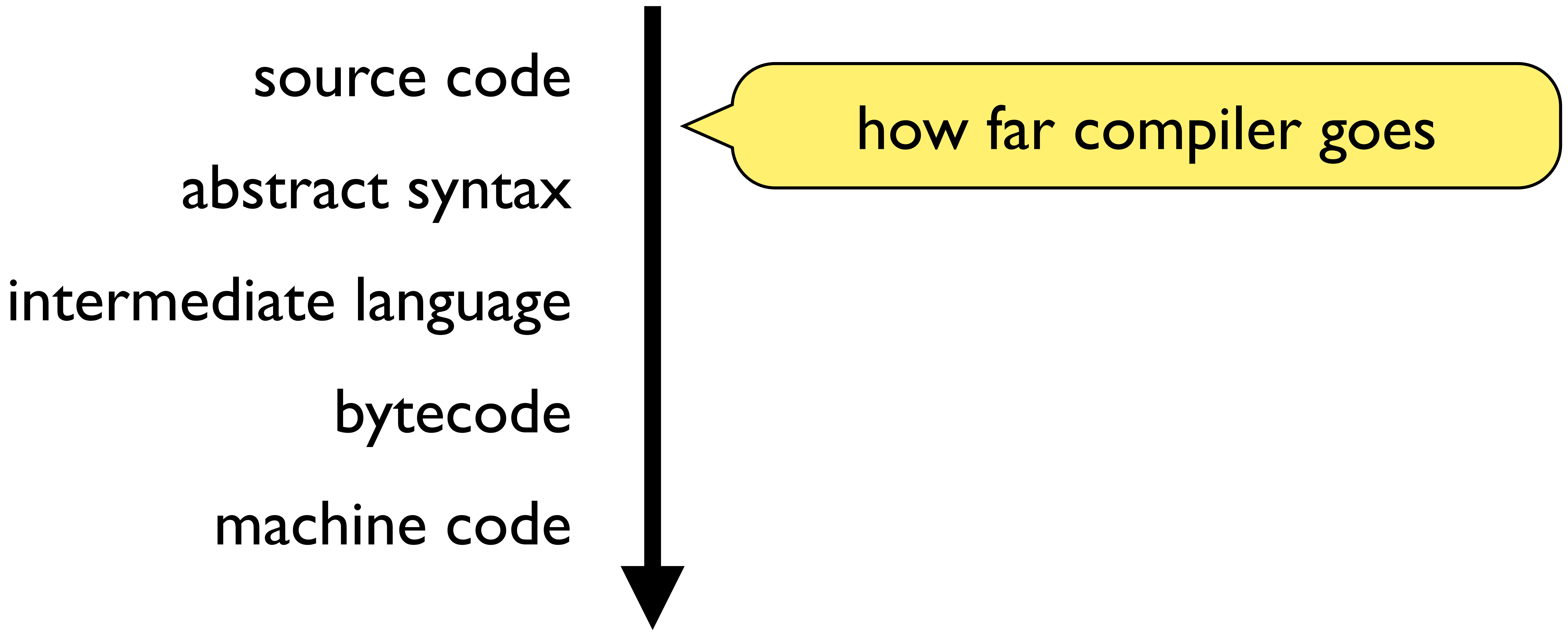

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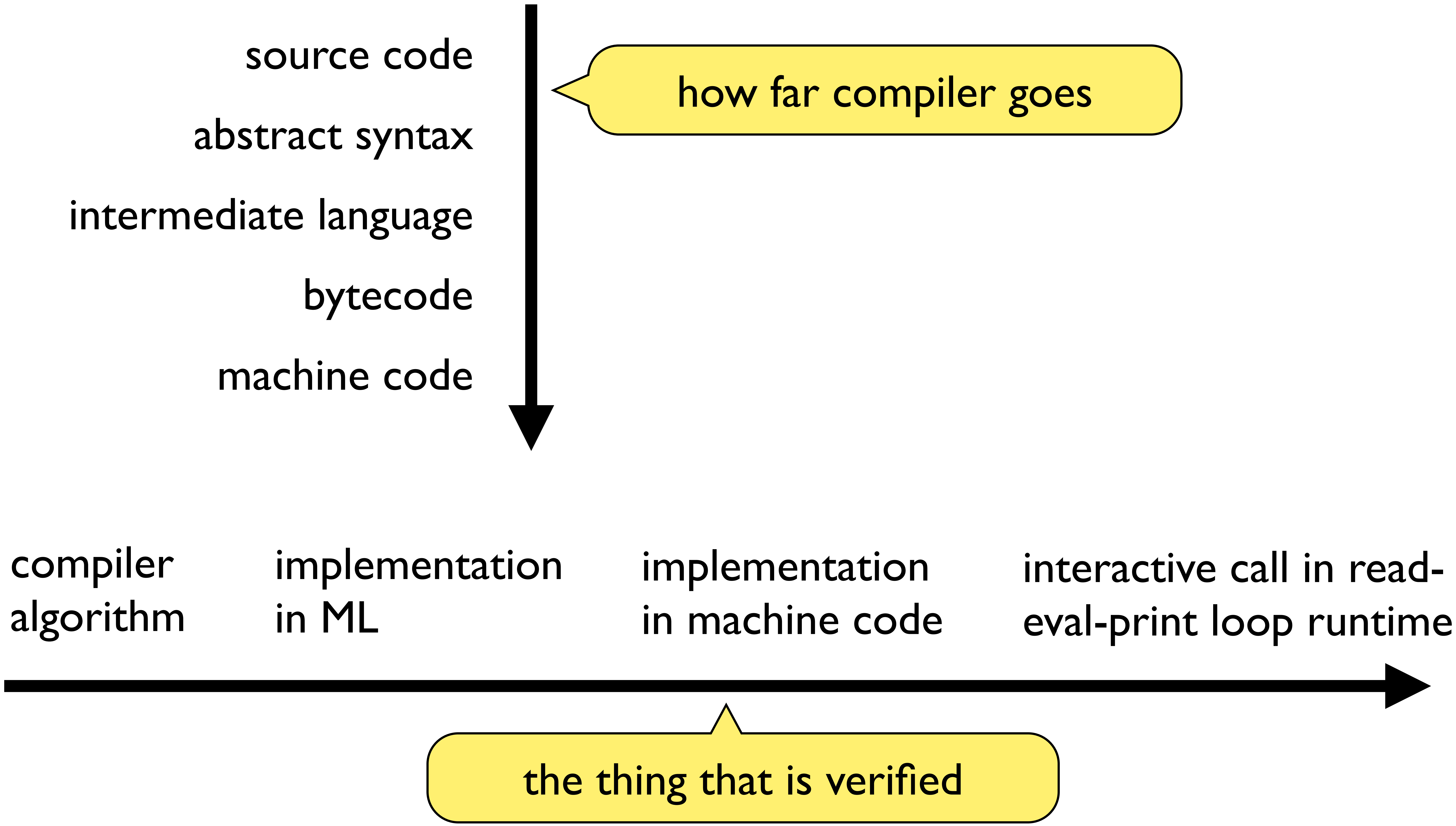

Dimensions of Compiler Verification

source code

abstract syntax

intermediate language

bytecode

machine code

how far compiler goes

Our verification covers the full spectrum of **both** dimensions.

compiler algorithm

implementation in ML

implementation in machine code

interactive call in read-eval-print loop runtime

the thing that is verified

Goal: End-to-end

$$\vdash P(\texttt{val x = ...})$$

 $[\![\cdot]\!]$

Ingredients

$$[\![\cdot]\!] : \texttt{string} \to v$$

Verified Compilation

$$[\![\cdot]\!] : \texttt{int list} \to s \to s$$

Bootstrapping

parsing

type inference

compilation

function in logic (compiler)

Bootstrapping

- parsing
- type inference
- compilation

function in logic (compiler)

↓ Proof-producing synthesis (ICFP'12)

CakeML program (compiler-ML)

⊢ compiler-ML implements compiler

Bootstrapping

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$\vdash$ compiler-ML implements compiler

by evaluation in the logic

$\vdash$ compiler (compiler-ML) = compiler-x86

Bootstrapping

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Proof-producing
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CakeML program (compiler-ML)

$\vdash$ compiler-ML implements compiler

by evaluation
in the logic

$\vdash$ compiler (compiler-ML) = compiler-x86

by compiler
correctness

$\vdash \forall c.$ (compiler c) implements c

Bootstrapping

- parsing
- type inference
- compilation

function in logic (compiler)

↓ Proof-producing synthesis (ICFP'12)

CakeML program (compiler-ML)

$\vdash$ compiler-ML implements compiler

by evaluation in the logic

$\vdash$ compiler (compiler-ML) = compiler-x86

by compiler correctness

$\vdash \forall c.$ (compiler c) implements c

Theorem: $\vdash$ compiler-x86 implements compiler

What Do These Words Mean?(III)

Photo by Kohei314, *via* `flickr.com`

- **Ecosystem:** Not only is the CakeML compiler "verified" (as before), but we also have a variety of methods for proving (other) CakeML programs correct.
- When your Haskell program misbehaves, who/what do you blame?
  - Your program (you)? GHC? The OS? The hardware? Cosmic rays?

Hmm, Can This Possibly Be True?

- Scepticism is fair.
- Must ask:
  - *"What are your assumptions?"*
- (Correct) Proofs are only as good as the assumptions behind them!

Assumption 1: our logic is sound.

- Any attempt to prove this would in turn depend on knowing that the logic being used to prove this was sound,
    - which would require another proof of soundness, carried out in yet another logical system…
        - this makes for an infinite regress…

- See also: Gödel.

from Wikipedia

Assumption 2: Our *implementation* of the logic is correct.

- HOL4 is *not* verified…
- The language it's written in hasn't been verified either
- *But*:
  - The *Trusted Code Base* in HOL4 is small by design
  - It's been eyeballed for many decades by experts
  - It can export proof logs for independent checking

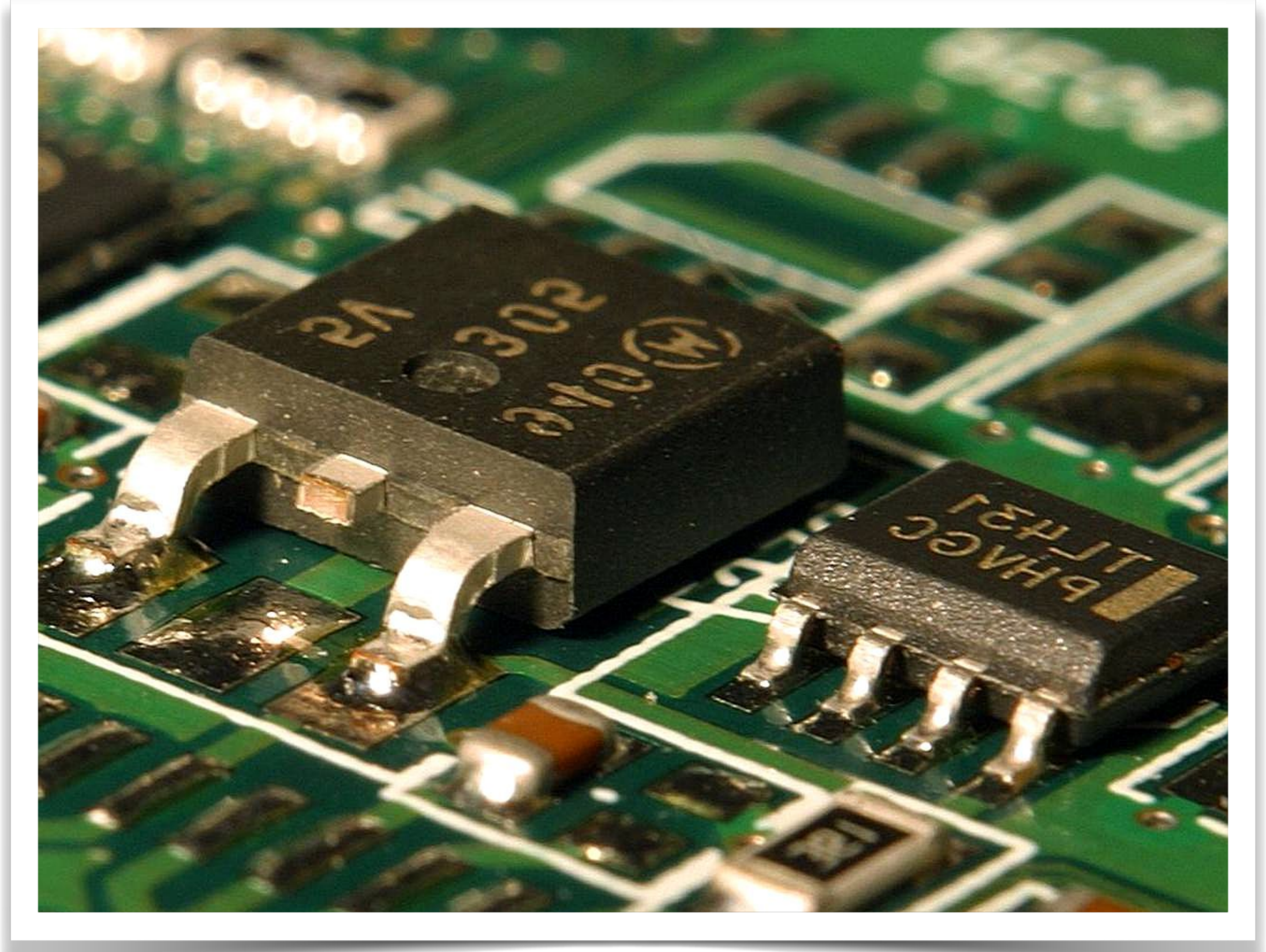
Assumption 3: Our correctness theorem says what we think it says

- Complicated logical statements are easy to misinterpret
- Luckily, our correctness statement is not *so* bad:

```
⊢ config_ok cc mc ⇒
   case compile cc prelude input of
    Success (bytes,ffi_limit) ⇒
     ∃ behaviours.
      cakeml_semantics ffi prelude input =
       Execute behaviours ∧
      ∀ ms.
       code_installed (bytes,cc,ffi,ffi_limit,mc,ms) ⇒
        machine_sem mc ffi ms ⊆
        extend_with_resource_limit behaviours
   | Failure ParseError ⇒
     cakeml_semantics ffi prelude input = CannotParse
   | Failure TypeError ⇒
     cakeml_semantics ffi prelude input = IllTyped
   | Failure CompileError ⇒ true
```

Assumption 4: Our logical model of the real world is accurate

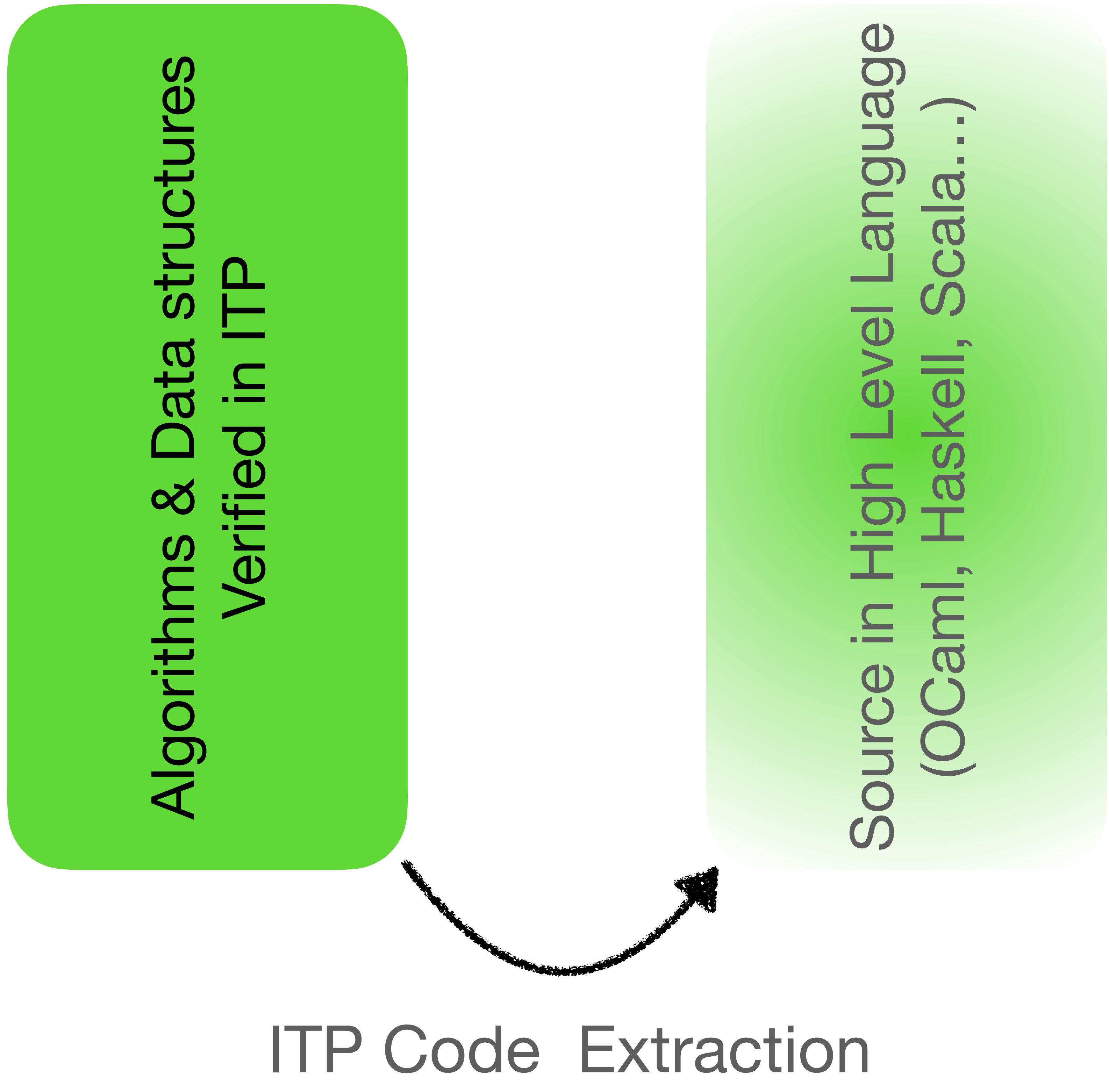
- We assume that x86 (ARM, RISCV,…) chips really do behave according to the logical spec we have for them.
- We assume that the OS implements its various system calls in accordance with our spec.



State-of-the-Art Assurance

Algorithms & Data structures
Verified in ITP

State-of-the-Art Assurance



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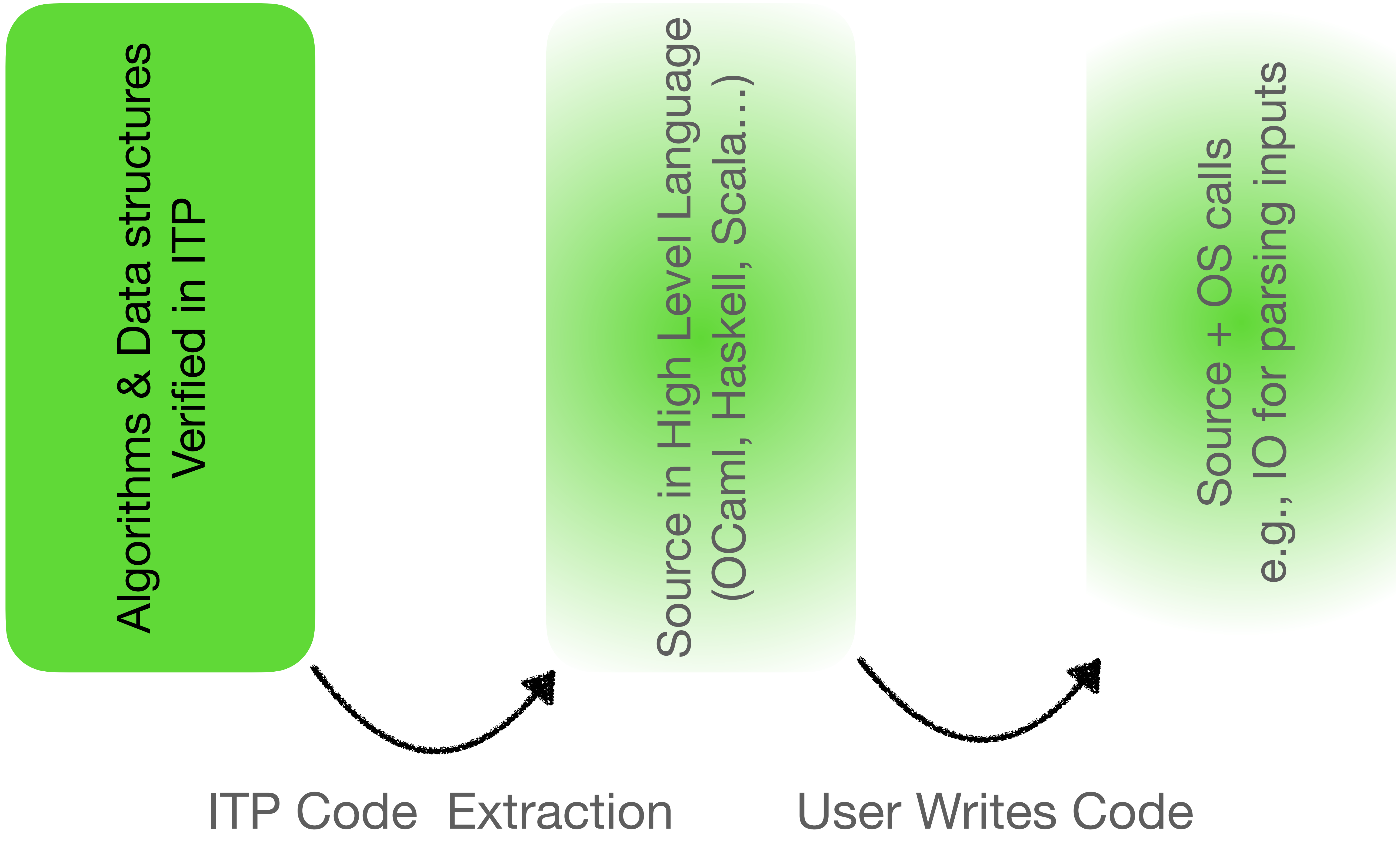

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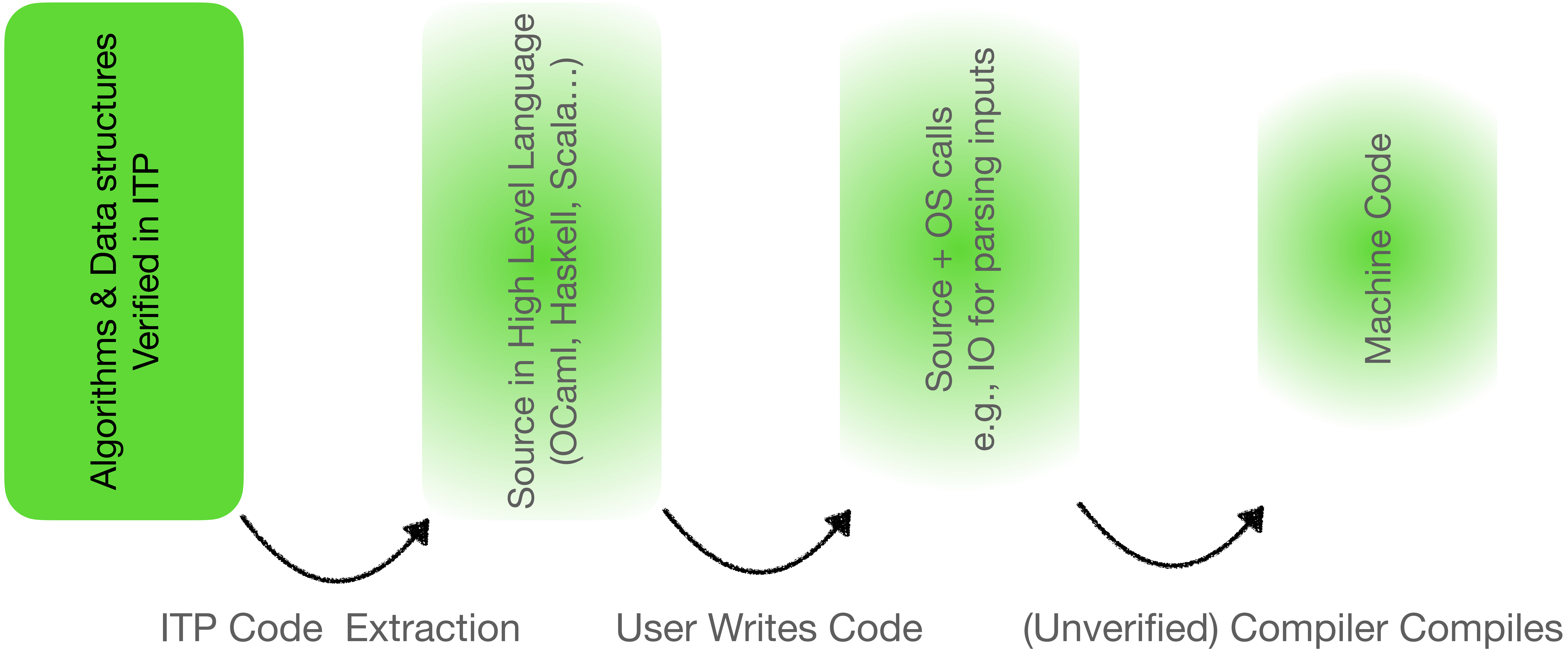
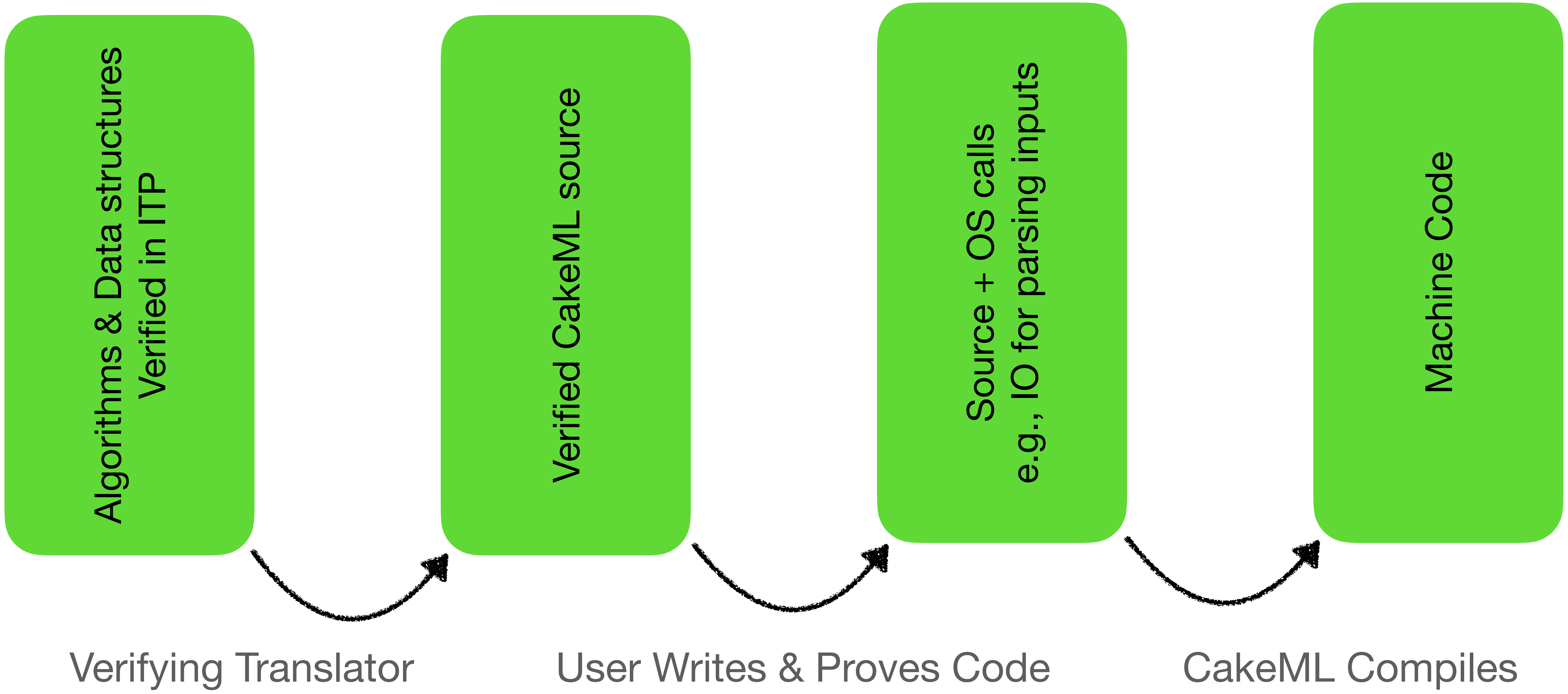


CakeML Assurance



CakeML Projects at Many Levels

CAKEML
A Verified Implementation of ML

Maths ⟷ Hardware

CakeML Projects at Many Levels

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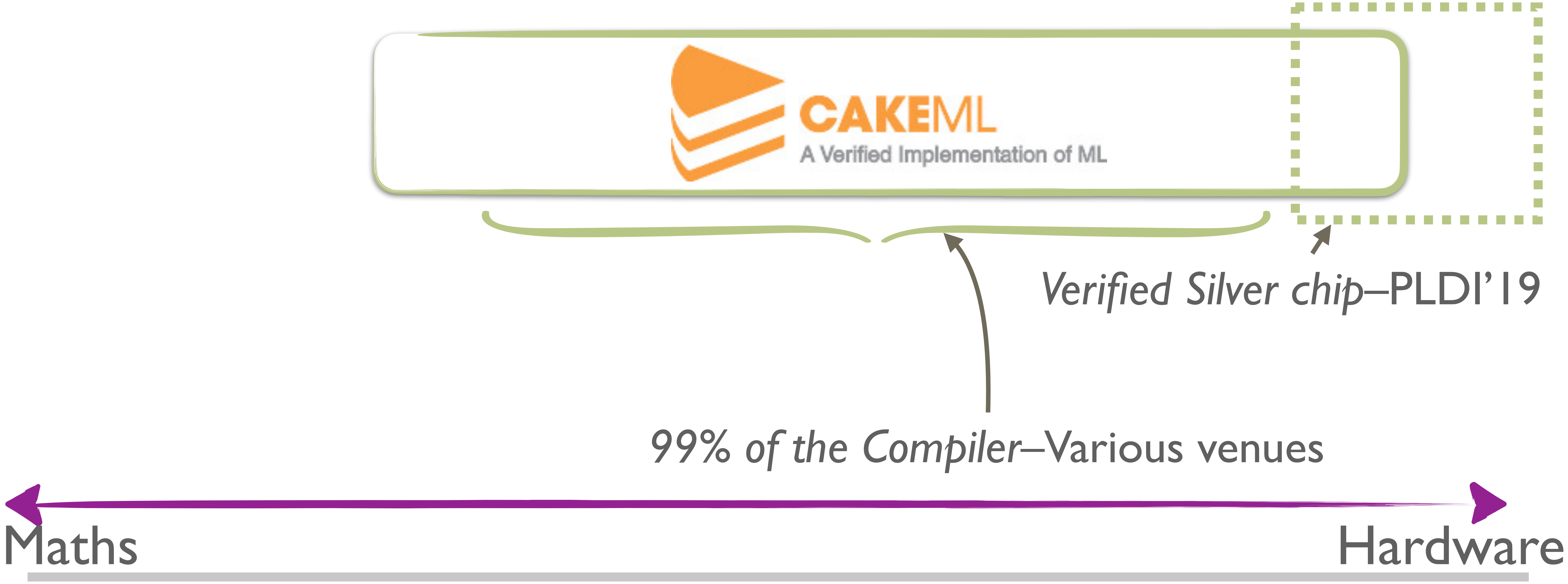

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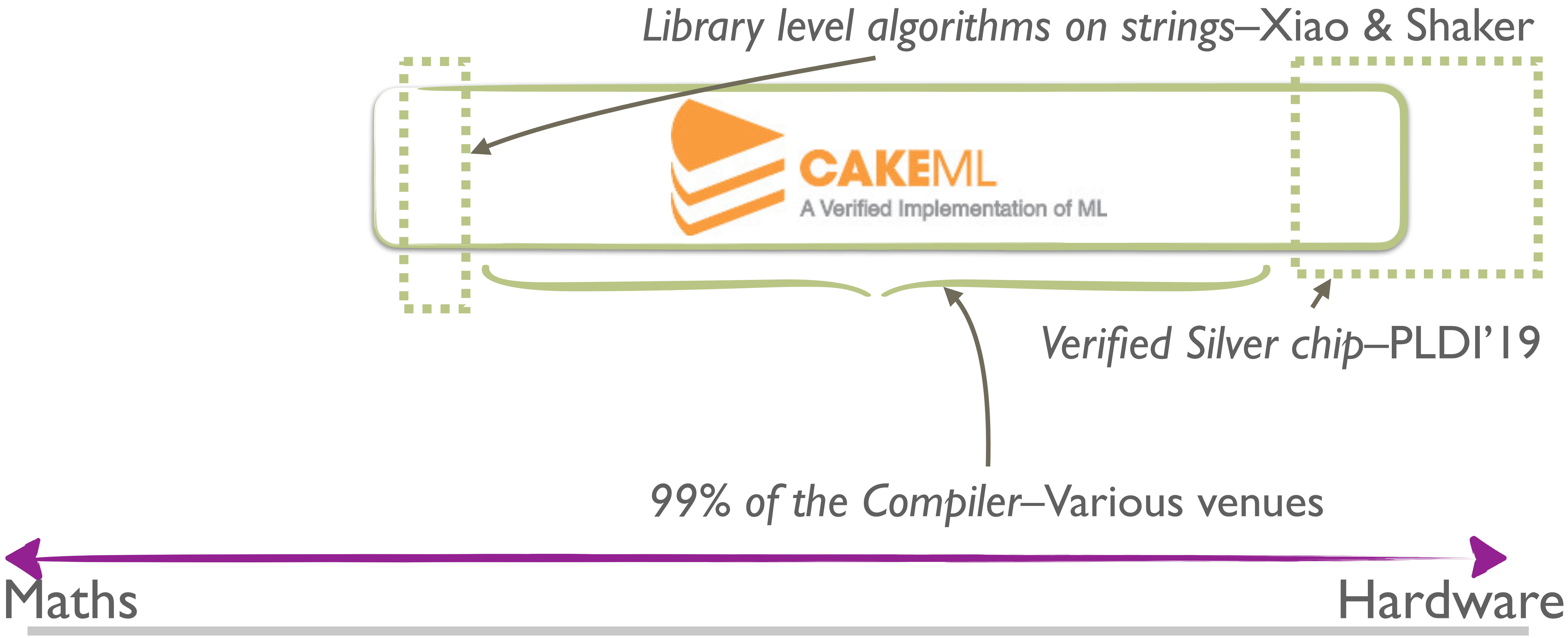

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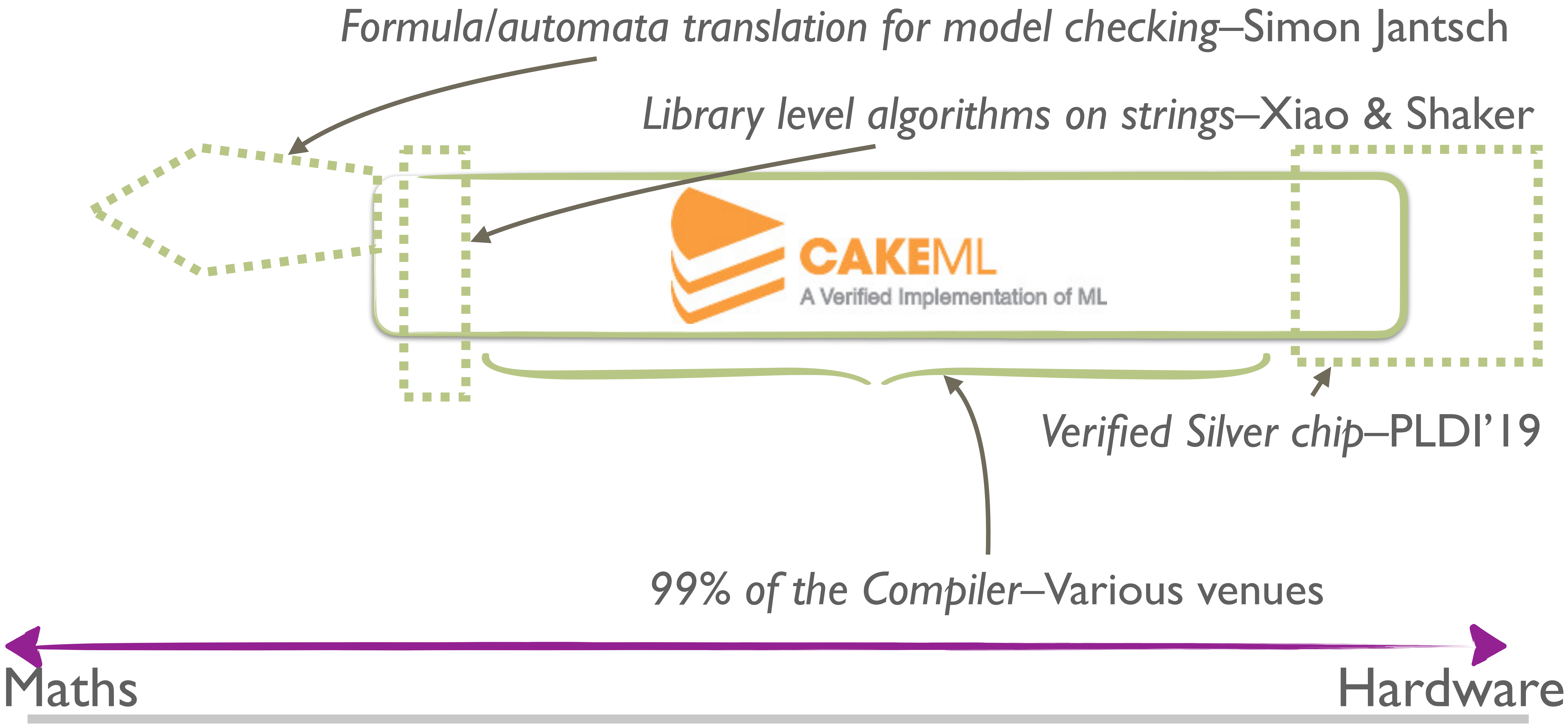

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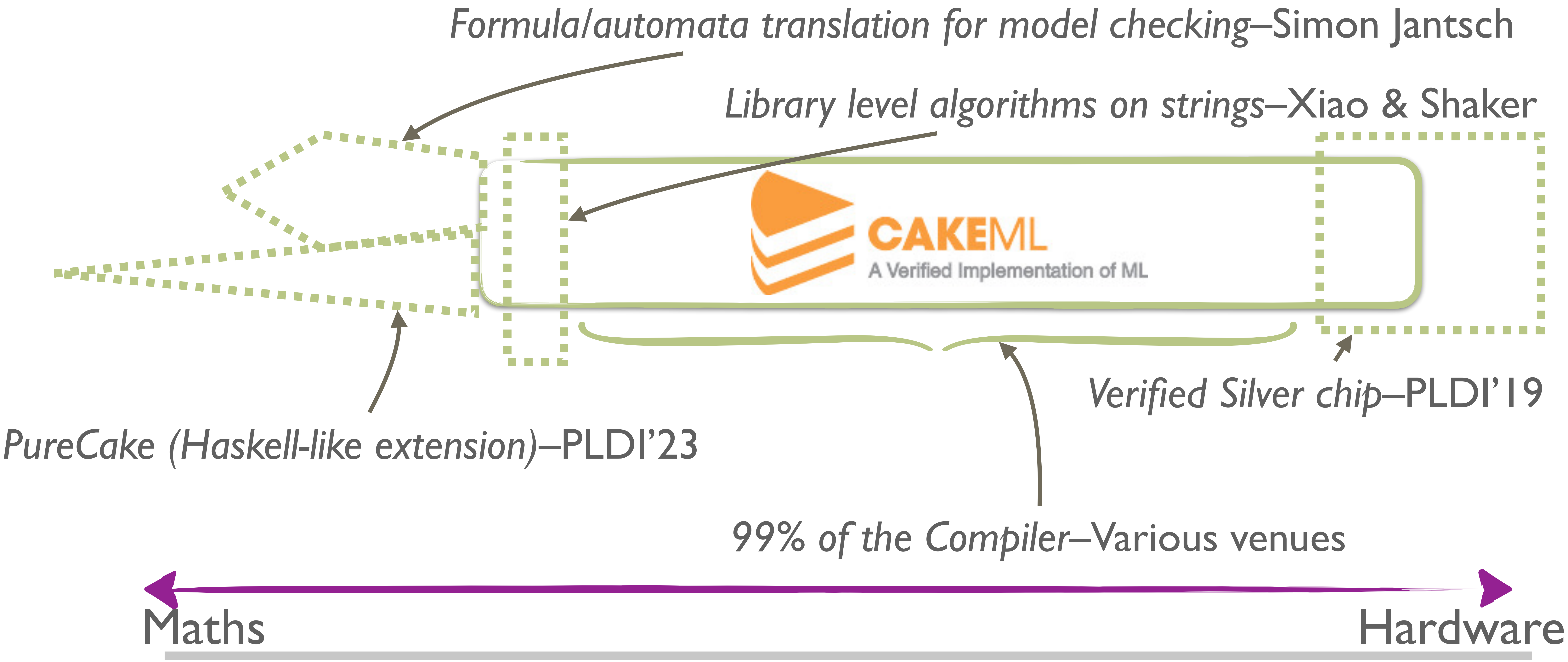

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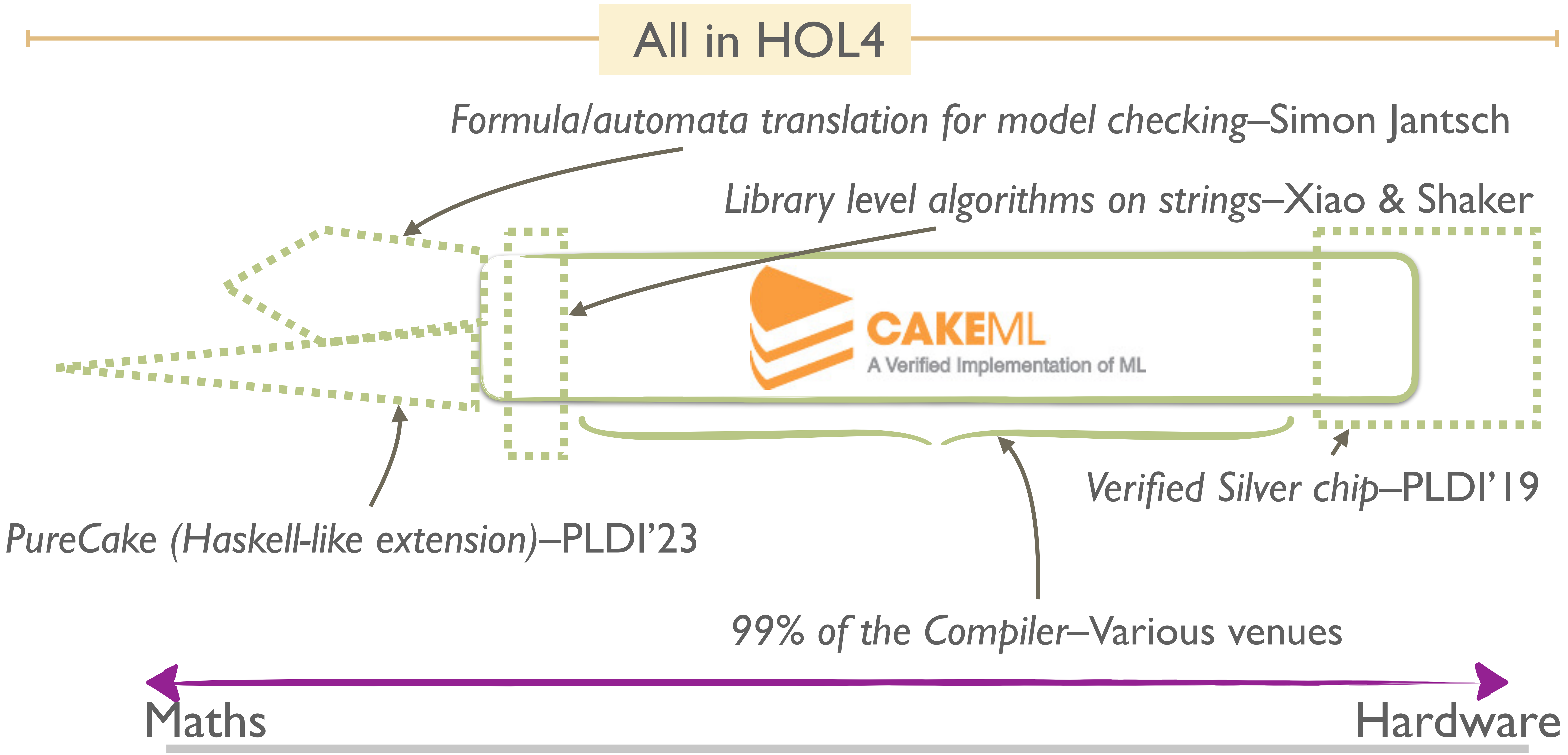
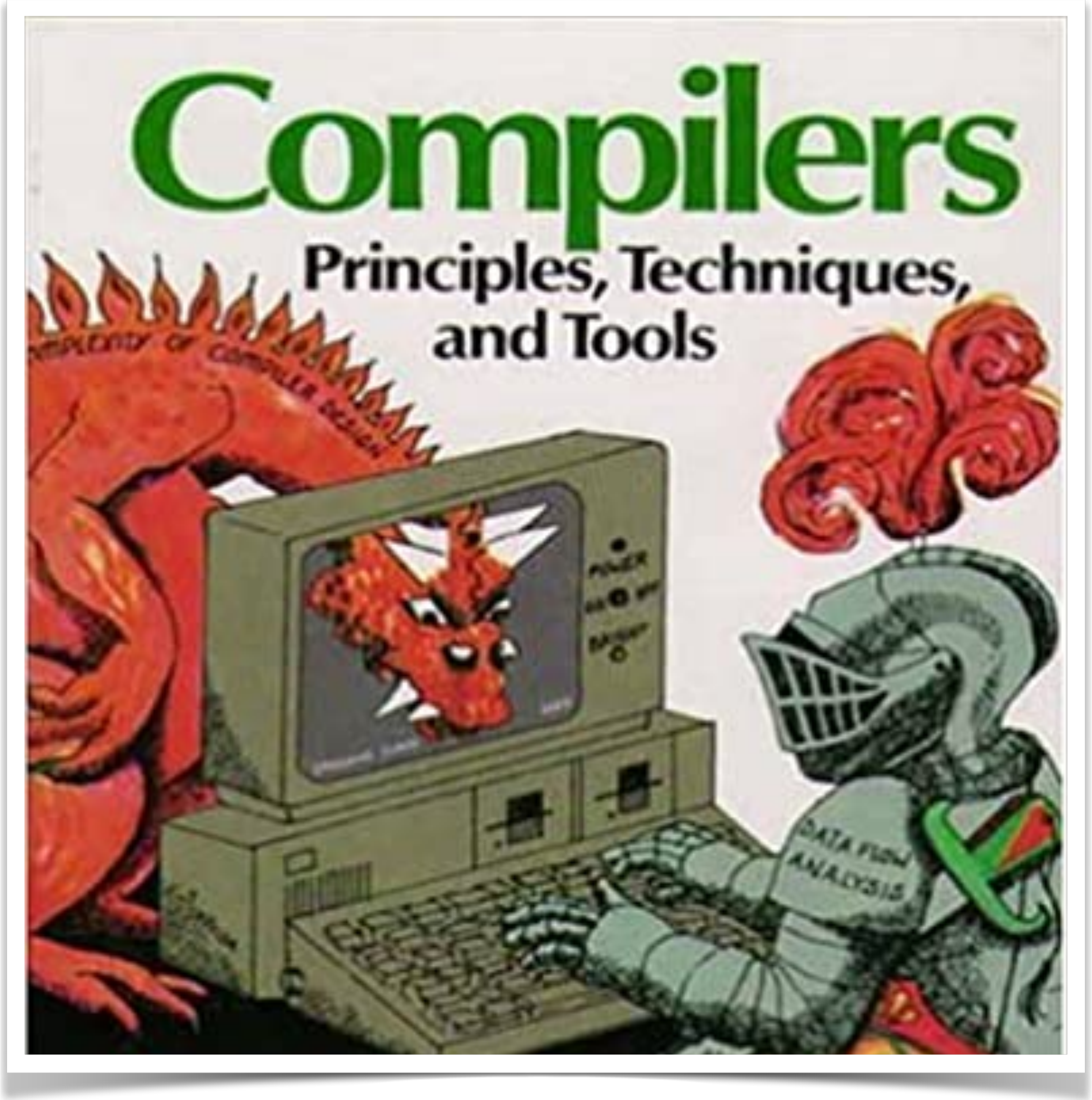
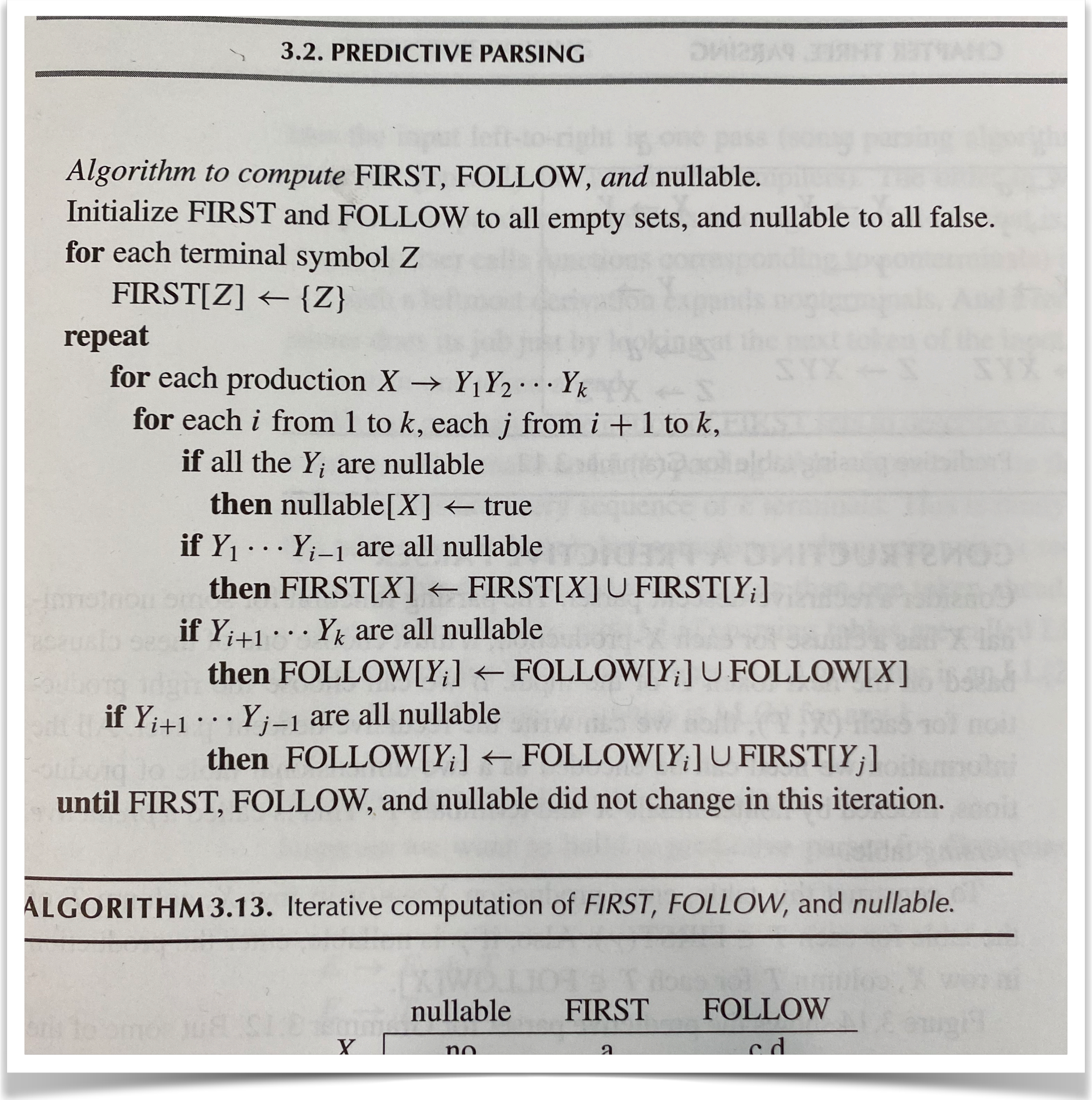


Parsing: an Application for the Ecosystem



- A verified, general-purpose, parser-construction tool is very appealing
- Applications (not just compilers) often need to parse input formats.
- “Verify Once, Run Ever-after”
  - Strong work in this area does already exist

Parsing: an Application for the Ecosystem


3.2. PREDICTIVE PARSING

Algorithm to compute FIRST, FOLLOW, *and* nullable.
Initialize FIRST and FOLLOW to all empty sets, and nullable to all false.
for each terminal symbol Z
    FIRST$[Z] \leftarrow \{Z\}$
repeat
    for each production $X \rightarrow Y_1 Y_2 \cdots Y_k$
        for each i from 1 to k, each j from $i+1$ to k,
            if all the Y_i are nullable
                then nullable$[X] \leftarrow$ true
            if $Y_1 \cdots Y_{i-1}$ are all nullable
                then FIRST$[X] \leftarrow$ FIRST$[X] \cup$ FIRST$[Y_i]$
            if $Y_{i+1} \cdots Y_k$ are all nullable
                then FOLLOW$[Y_i] \leftarrow$ FOLLOW$[Y_i] \cup$ FOLLOW$[X]$
        if $Y_{i+1} \cdots Y_{j-1}$ are all nullable
                then FOLLOW$[Y_i] \leftarrow$ FOLLOW$[Y_i] \cup$ FIRST$[Y_j]$
until FIRST, FOLLOW, and nullable did not change in this iteration.

ALGORITHM 3.13. Iterative computation of *FIRST, FOLLOW,* and *nullable.*

	nullable	FIRST	FOLLOW
X	no	a	c d

from: Appel, *Modern Compiler Implementation in ML*

- CakeML's existing parser is a custom-built PEG
- Its verification was just as "custom" (*i.e.*, tedious)
- General tools need general treatments of things like *first* and *follow* sets

Grammars, Classically

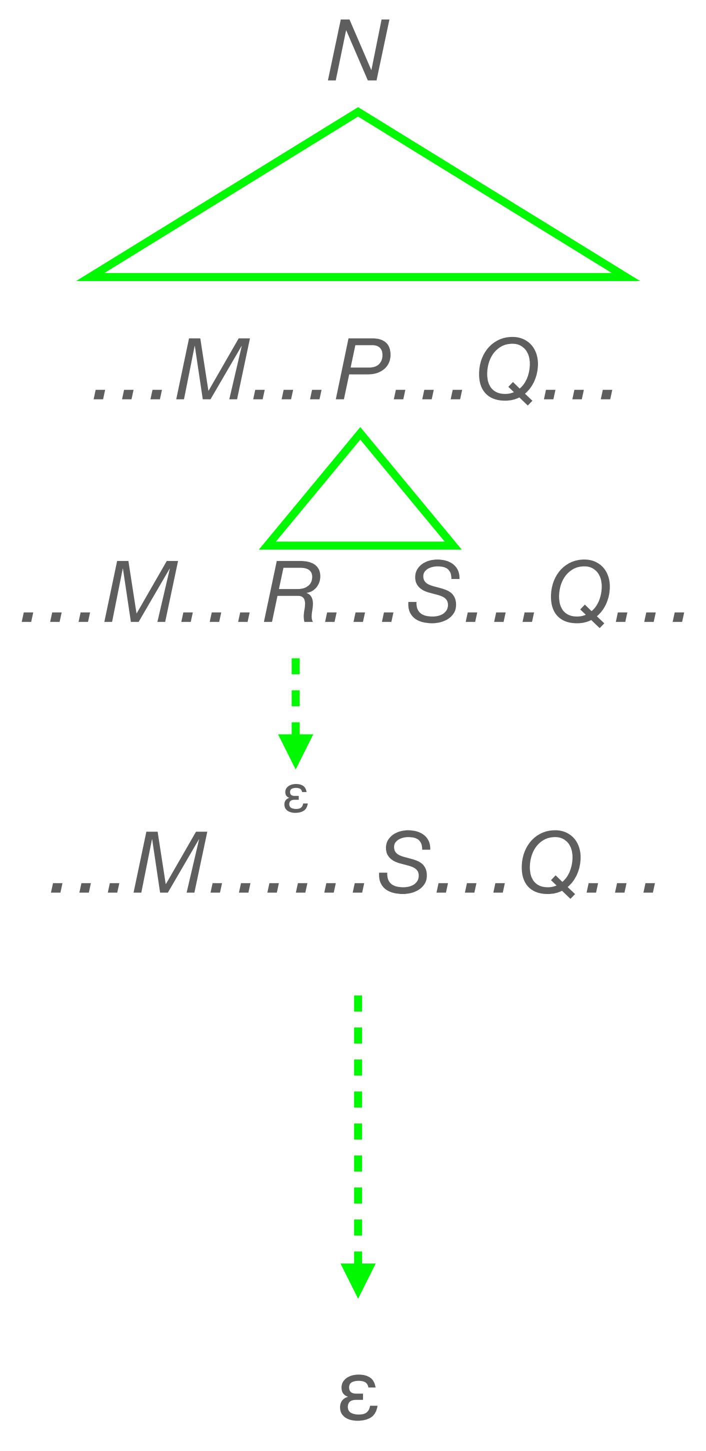
A grammar is a 4-tuple (G, N, T, S), with

- N a finite set of non-terminal symbols;
- T a finite set of terminal symbols;
- $S \in N$ a distinguished non-terminal (the "start symbol");
- G a finite set of production rules, each of the form: $N \rightarrow (N+T)^*$

"I never met a finite set I didn't want to treat as a list"

–Every interactive theorem-proving person ever

Calculating Nullability

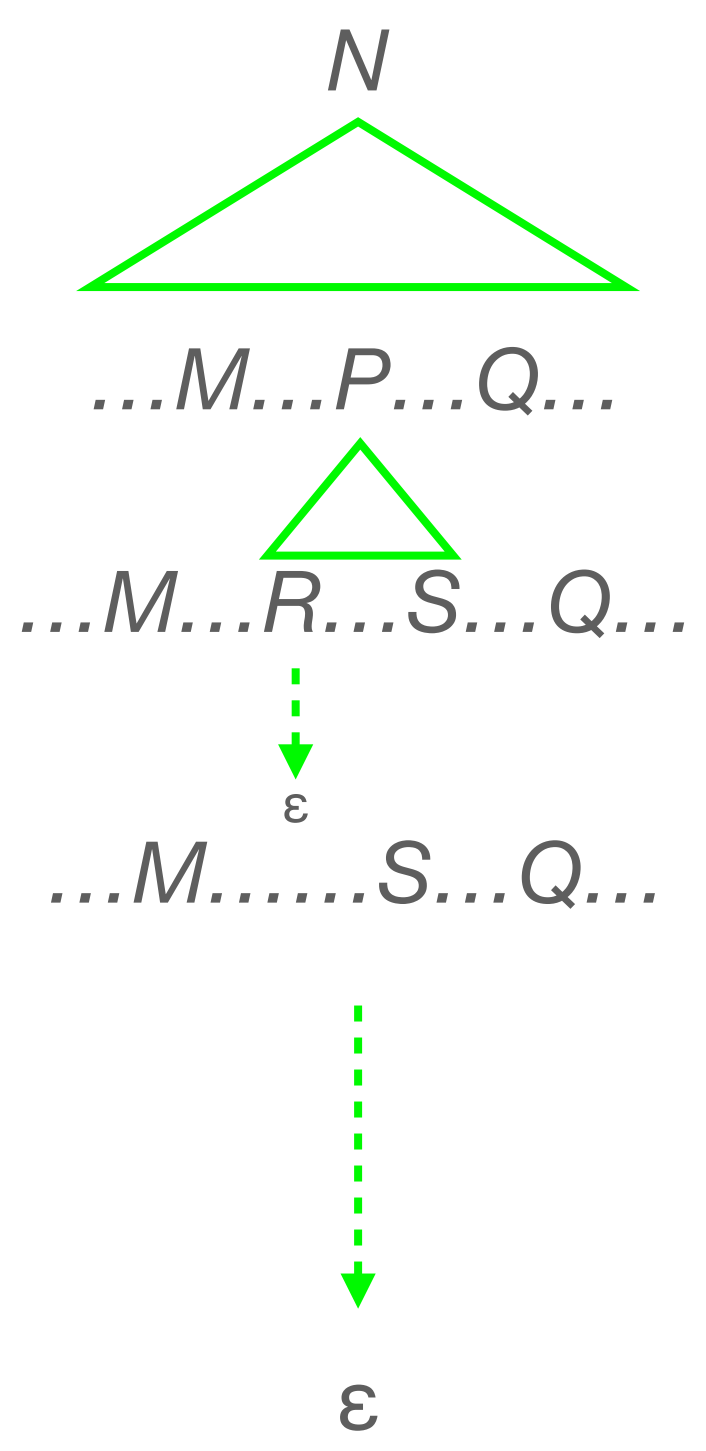


Start with the mathematical definition:

Definition nullable_def:
nullable G sf ⇔ derives G sf [ ]
End

Where *derives* is the reflexive and transitive closure of the relation that expands a non-terminal into a production rule's RHS.

Calculating Nullability



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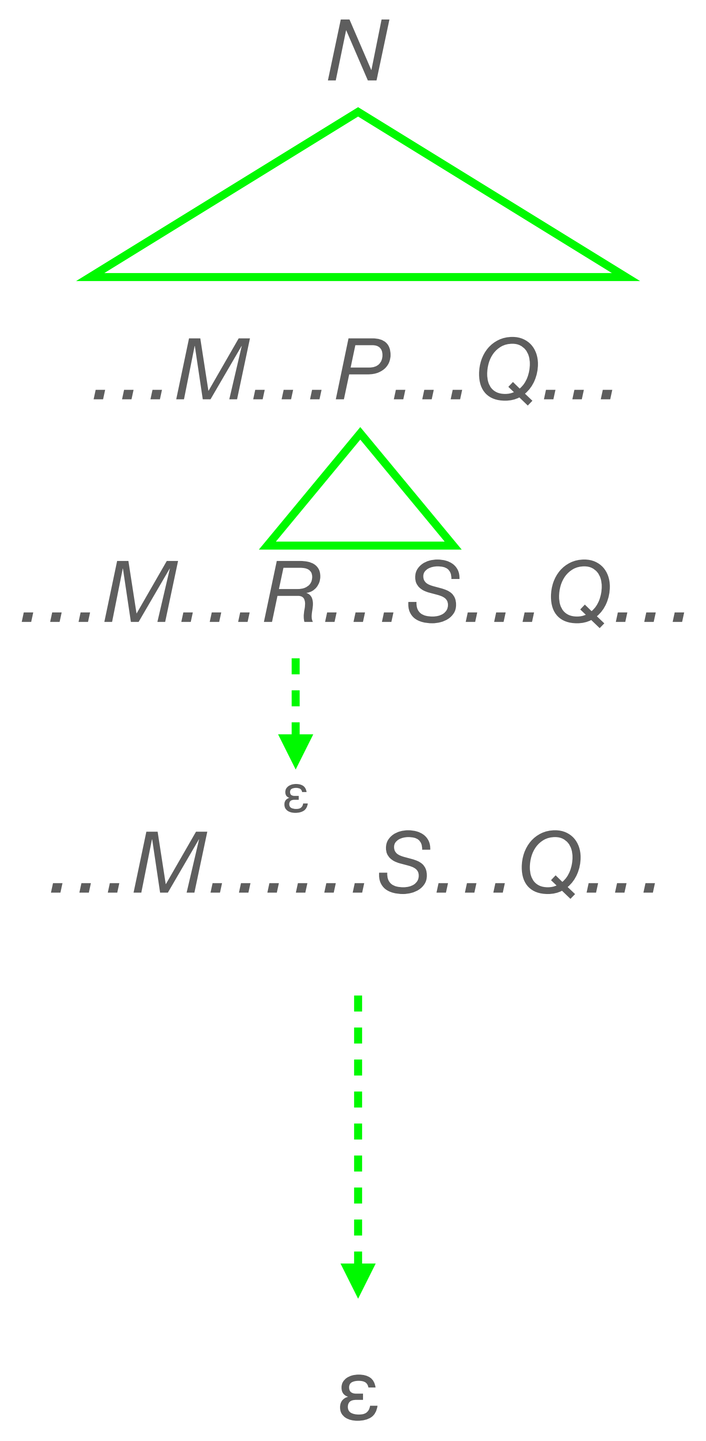

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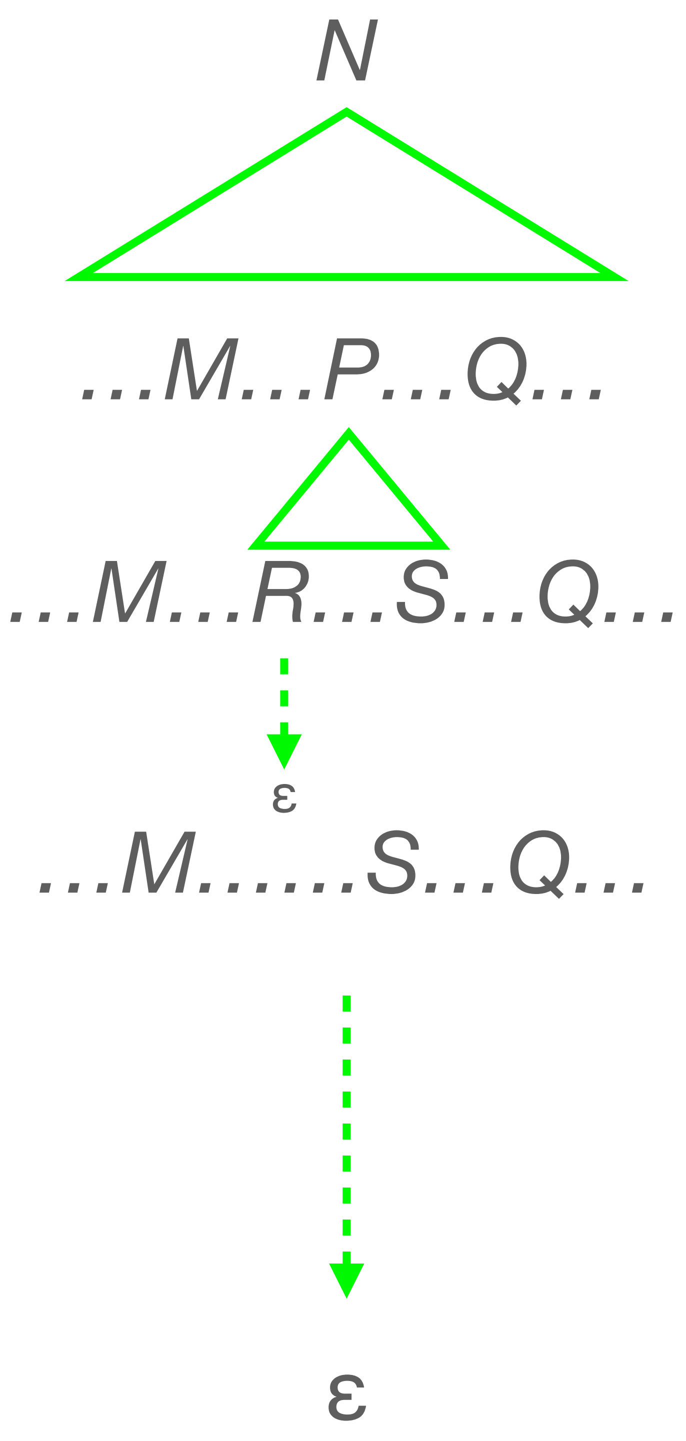
End

Where *derives* is the reflexive and transitive closure of the relation that expands a non-terminal into a production rule's RHS.

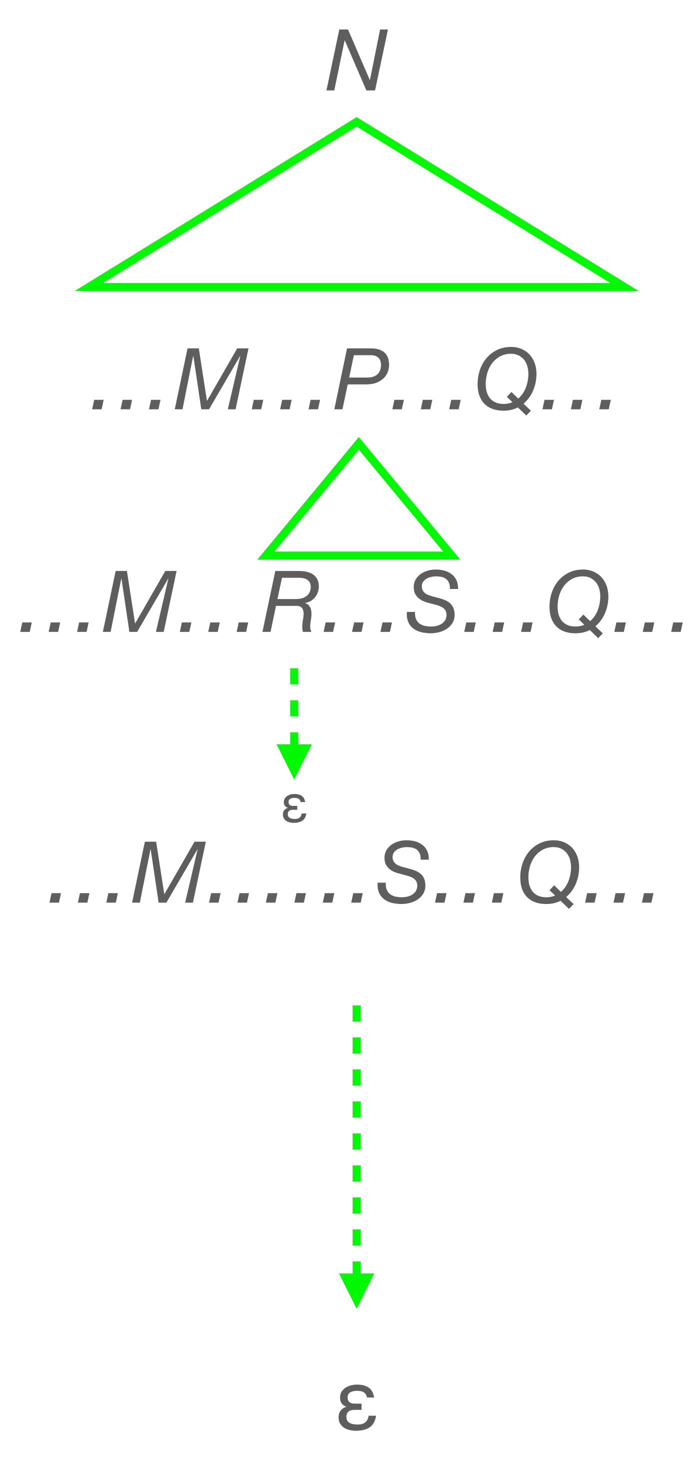
- Terminals are **not** nullable.
- Non-terminals are nullable if any of their RHSs are nullable.
- Critical Realisation: recursive loopbacks can be ignored.

Calculating Nullability



Recursive algorithm:

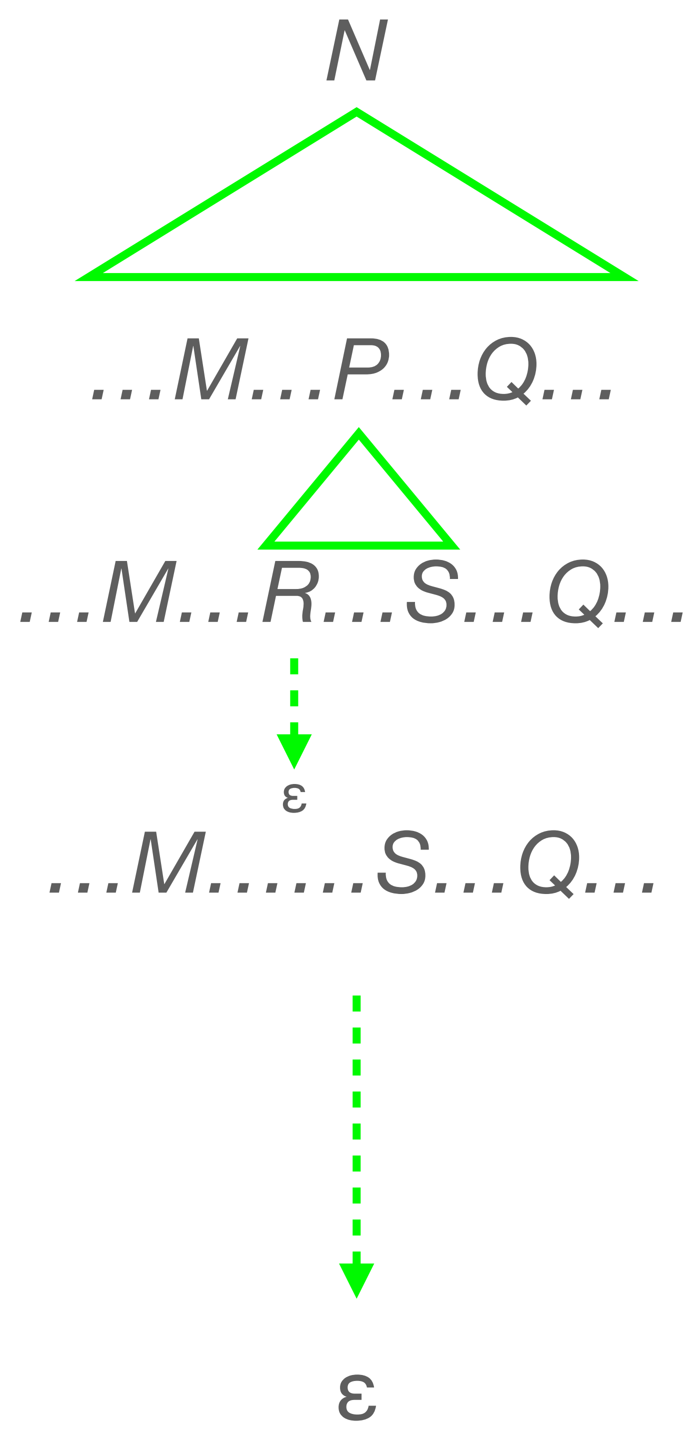
Calculating Nullability



Recursive algorithm:

```
nullableA G s [ ] = T
```

Calculating Nullability

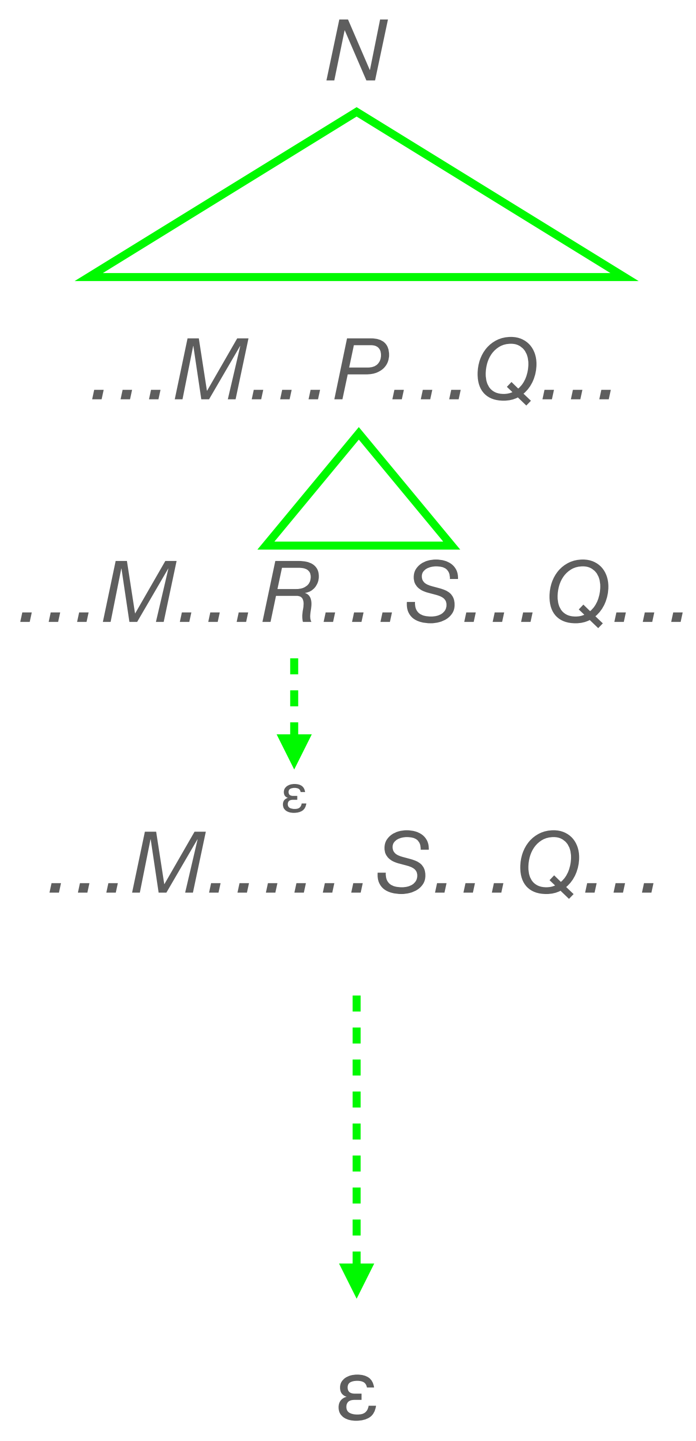


Recursive algorithm:

```
nullableA G s [ ] = T

nullableA G s (TOK _ :: _) = F
```

Calculating Nullability



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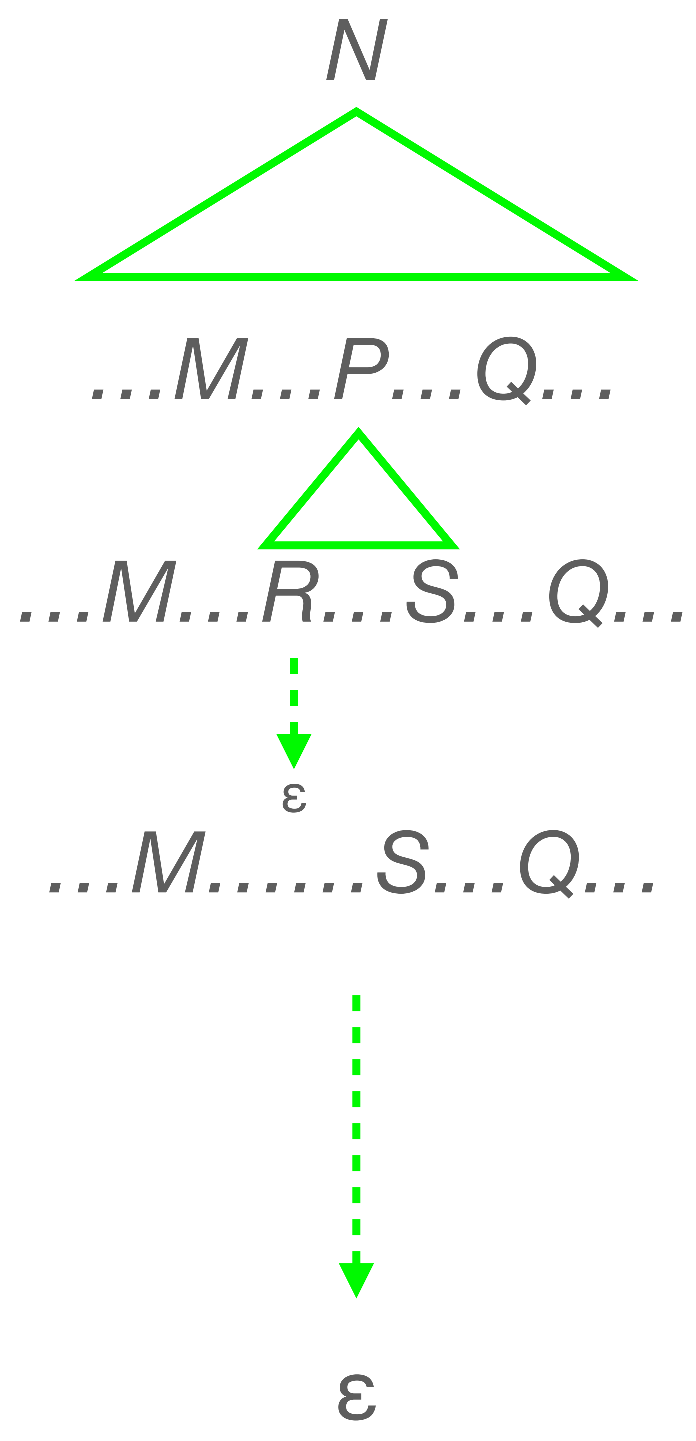
```

Calculating Nullability



Recursive algorithm:

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nullableA G s (NT n :: rest) =
    nullableA G s rest ∧
```
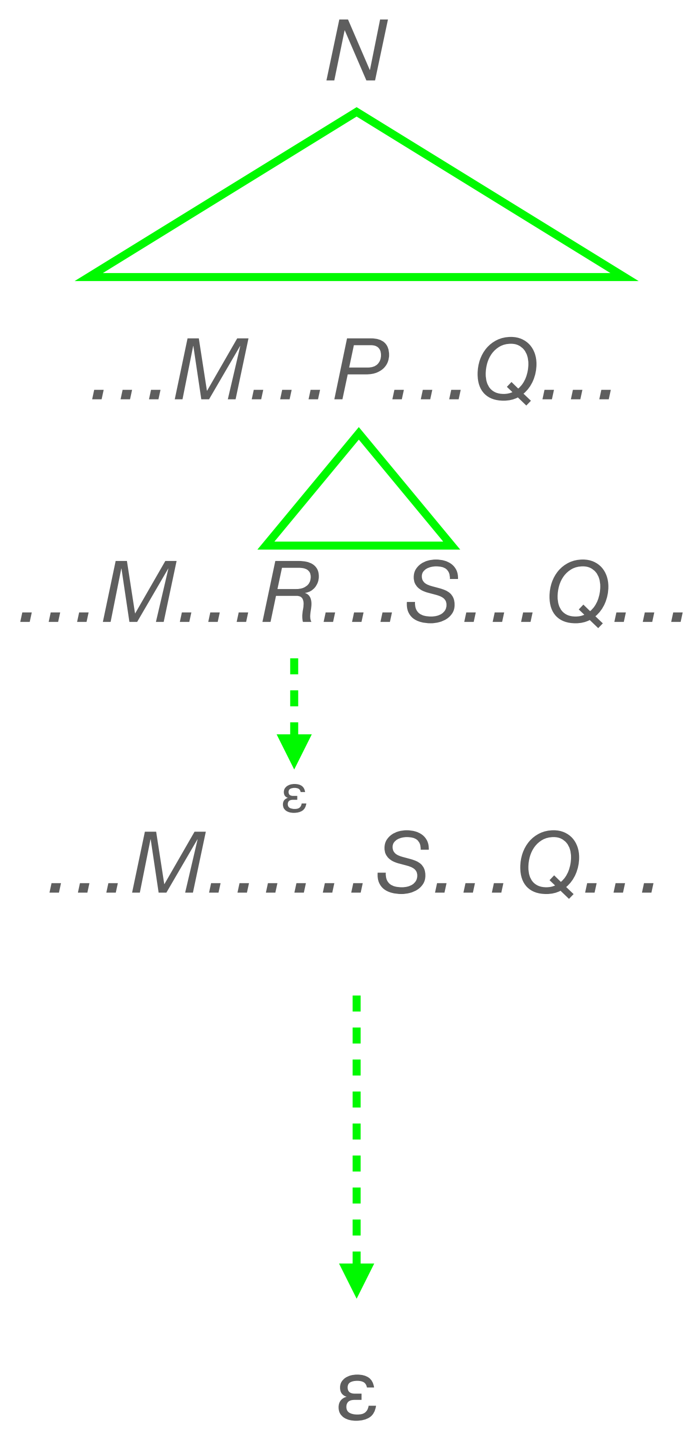

Calculating Nullability



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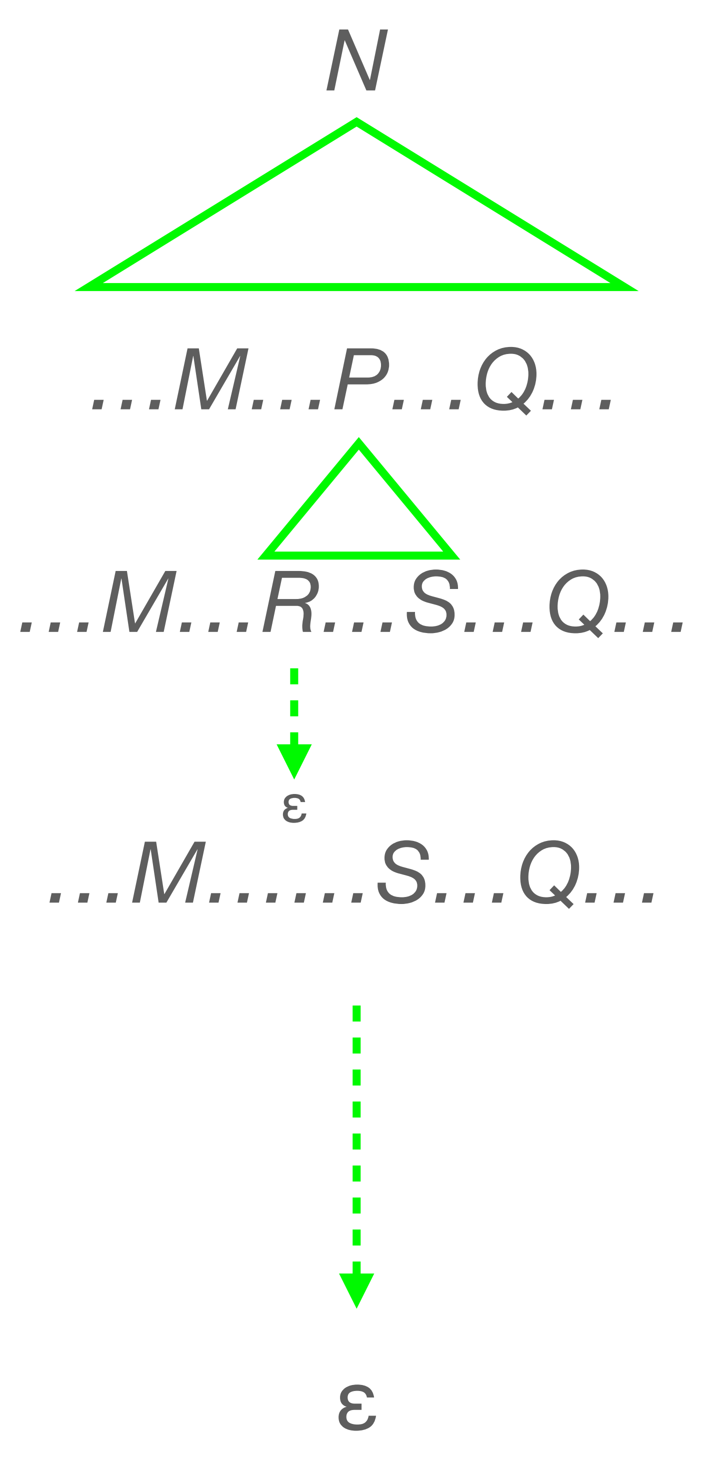
     n  is not a member of set s   ∧
```

Calculating Nullability



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     n is not a member of set s   ∧
     nullableA G (n INSERT s) r
          For some r a production for non-terminal n
```
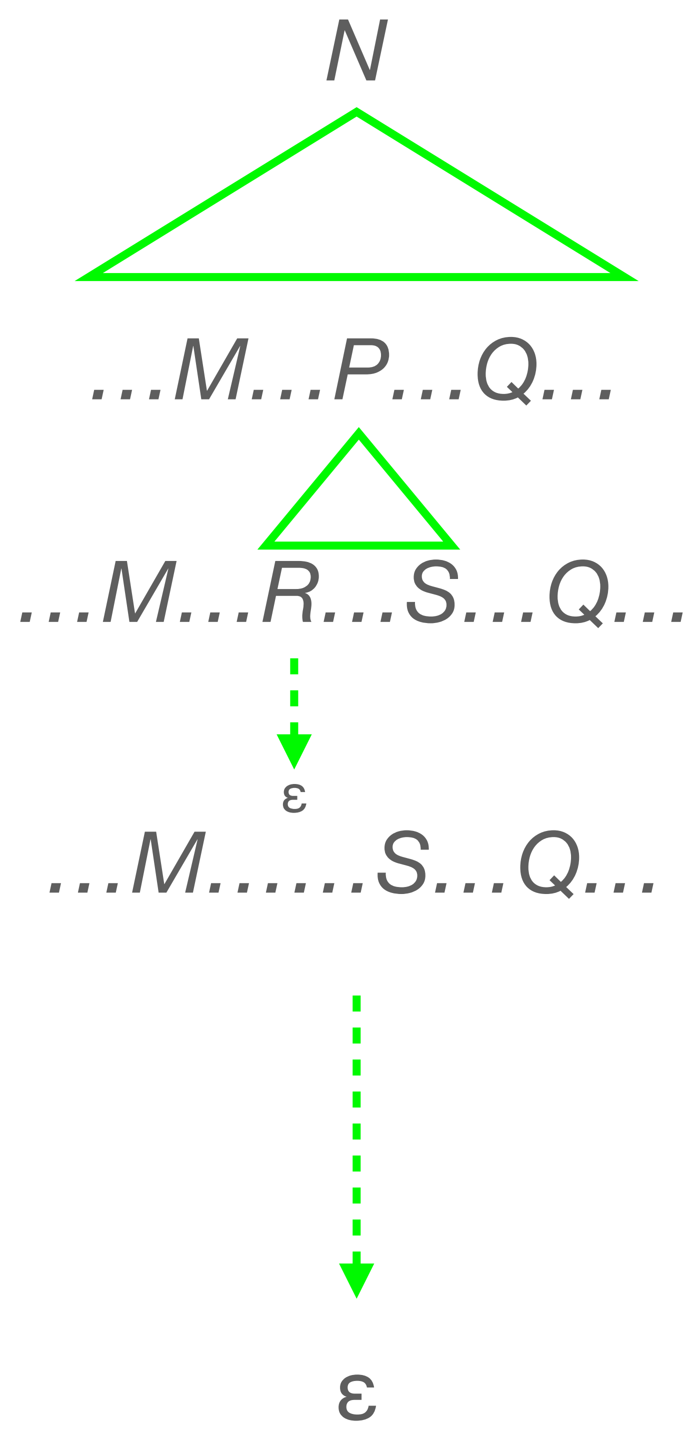

Calculating Nullability



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Theorem:

nullable G sf ⇔ nullableA G ∅ *sf*

Clean, Mathematical Formulations

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A high-level property characterising *nullability* can be re-expressed more "algorithmically"

- without using lists!

Clean, Mathematical Formulations

A high-level property characterising *nullability* can be re-expressed more "algorithmically"

- without using lists!

The notion of *first* set can be handled similarly:

- A sentential form has a *first* set (just as an s.f. may be nullable)
- Uses "seen" set of visited non-terminals (recursive calls can be ignored)

Iterating Over all of a Grammar

Formulations of *nullable* and *first* are functions on sentential forms.

Each of a grammar's non-terminals are themselves (short) sentential forms.

Thus: we can take the image of these functions over the non-terminal set, and be done.

- Computationally, this looks bad: calculating e.g., *nullable(N)* will recalculate *nullable* for all non-terminals *N* refers to, and so on, recursively.

Essence of Refinement

CENEX oil refinery, Montana—Greg Goebel *via* flickr.com

- Haven't committed to using lists to represent grammars
- Have separated concerns
- Have deferred other algorithmic decisions
- Have already lost some efficiencies…

The Evil That Is the Follow Set

The "*iterate until result stops changing*" seems unavoidable.

It's also painful:

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It's also painful:

> Because of this algorithm's "iterate until convergence" structure, we need to do some extra work to prove that it terminates. To accomplish this task, we use Coq's Program extension [18], which provides support for defining functions using well-founded recursion. The `Program Fixpoint` command enables the user to define a non-structurally recursive function by providing a measure—a mapping from one or more function arguments to a value in some well-founded relation $\mathcal{R}$—and then showing that the measure of recursive call arguments is less than that of the original arguments in $\mathcal{R}$.

—Lasser, Casinghino, Fisher, Roux (ITP'2019)

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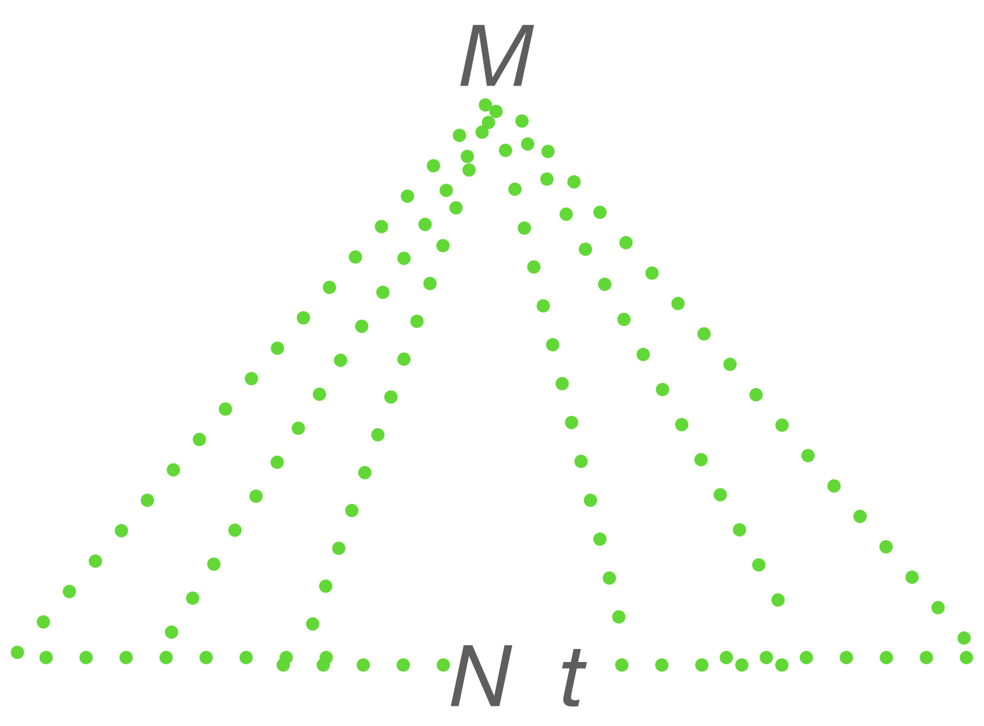
(The paper above is following Appel and doing this for all of *nullable*, *first*, and *follow*.)

Follow's Clean Characterisation (I)



- Symbol *t* is in *N*'s follow set if there is a valid derivation from some *M* ending in a sentential form with *t* occurring immediately after *N*.
- The "all at once" view

Follow's Clean Characterisation (II)

The equivalent step-at-a-time view, following Lasser *et al.*:

$$\frac{M \to \alpha N \beta \ \in \ G \qquad a \in \mathsf{first}_G(\beta)}{a \in \mathsf{follow}_G(N)}$$

$$\frac{M \to \alpha N \beta \ \in \ G \qquad \mathsf{nullable}_G(\beta) \qquad a \in \mathsf{follow}_G(M)}{a \in \mathsf{follow}_G(N)}$$

Here, the recursive reference is “backwards” (which rules does *N* appear *in*), and recursions can’t be ignored.

Iterating Over Finite Sets

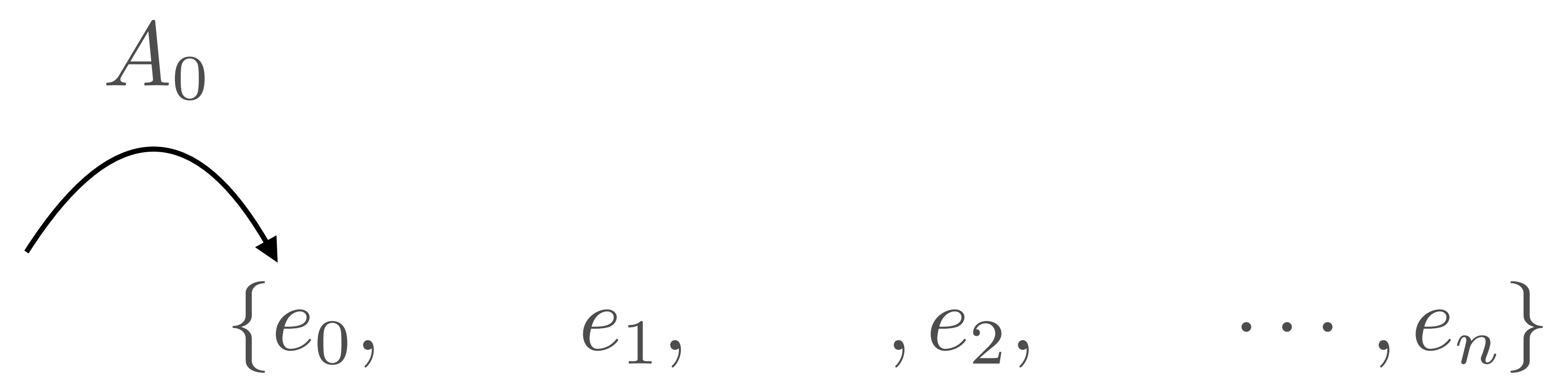
$\{e_0, \quad e_1, \quad , e_2, \quad \cdots, e_n\}$

Iterating Over Finite Sets

$\{e_0, \quad e_1, \quad , e_2, \quad \cdots , e_n\}$

- We want a fold-like way to iterate over the elements of the set (e.g., grammar's rules).
- (Making the set look like a list.)

Iterating Over Finite Sets



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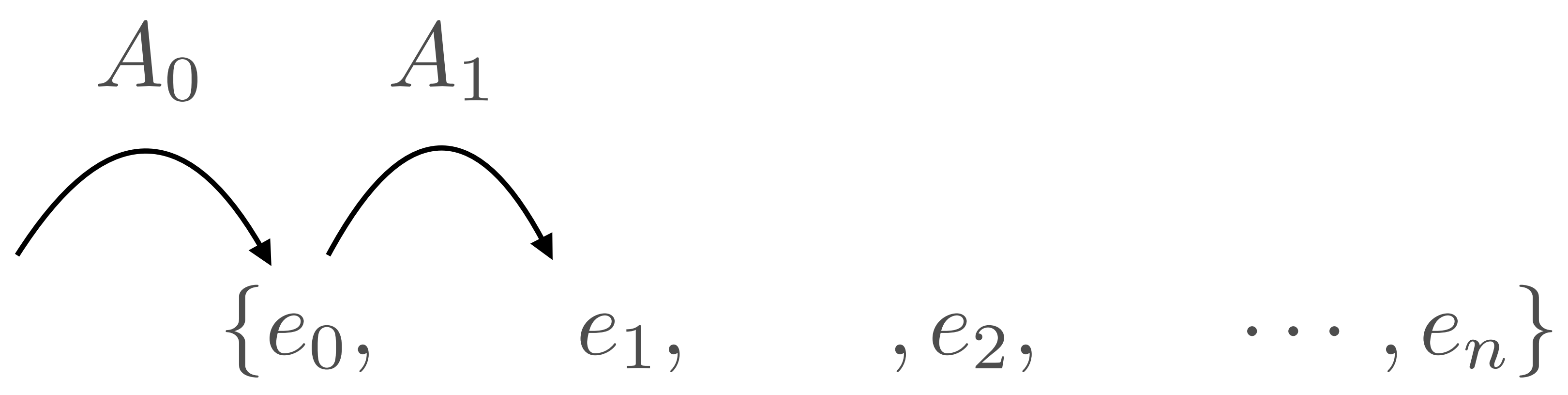

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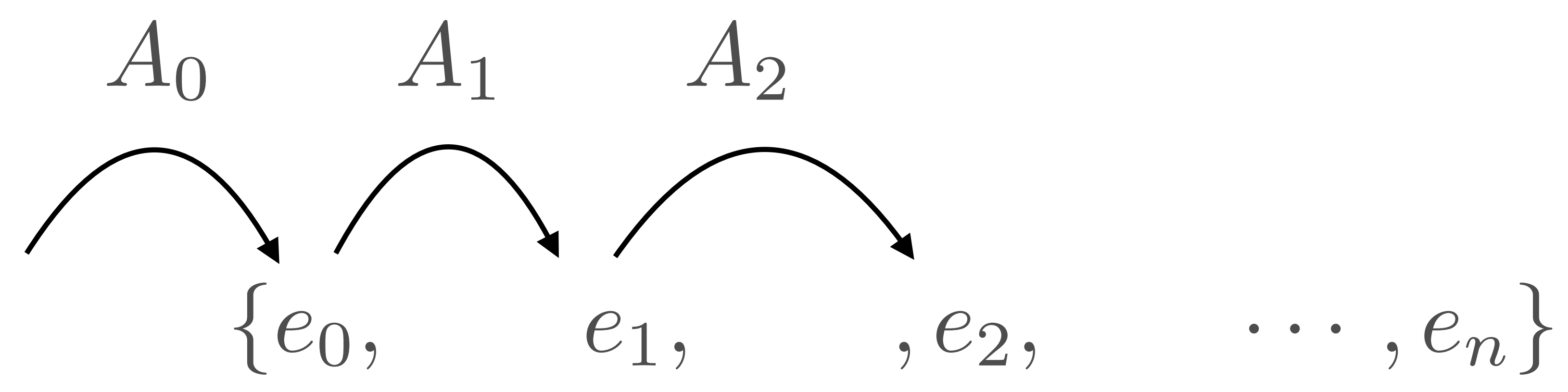

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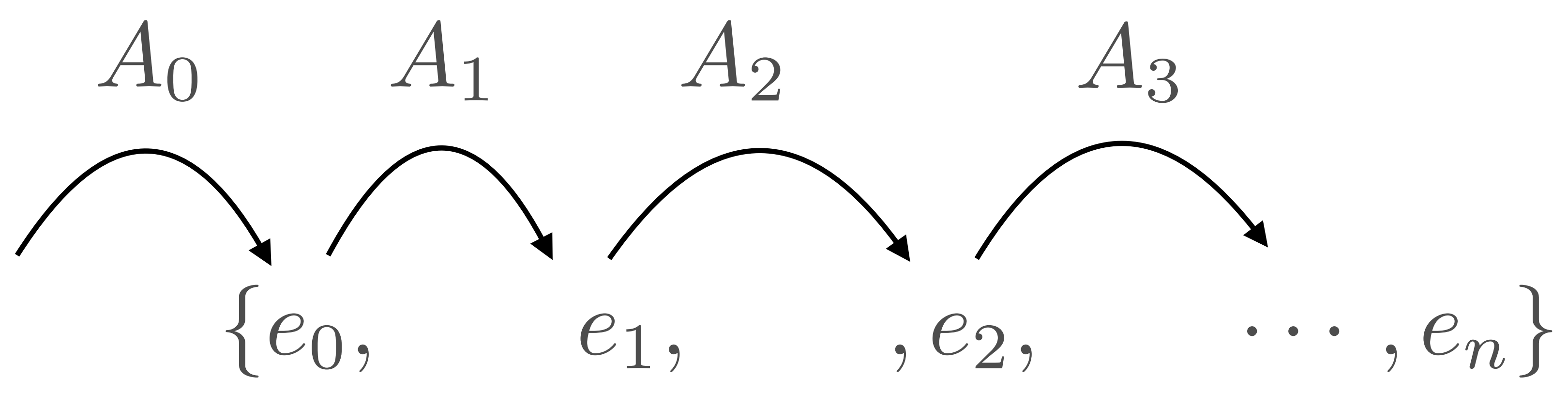
Iterating Over Finite Sets



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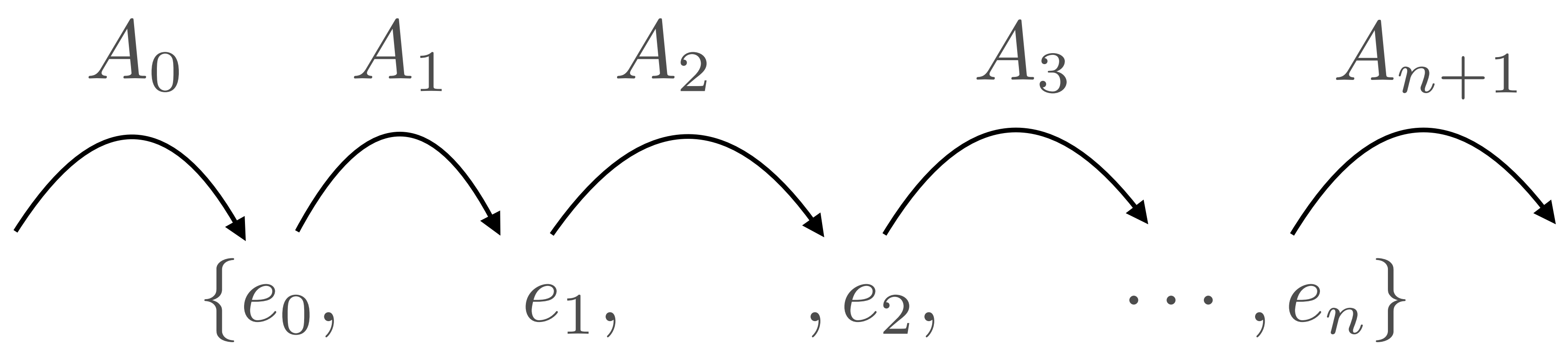

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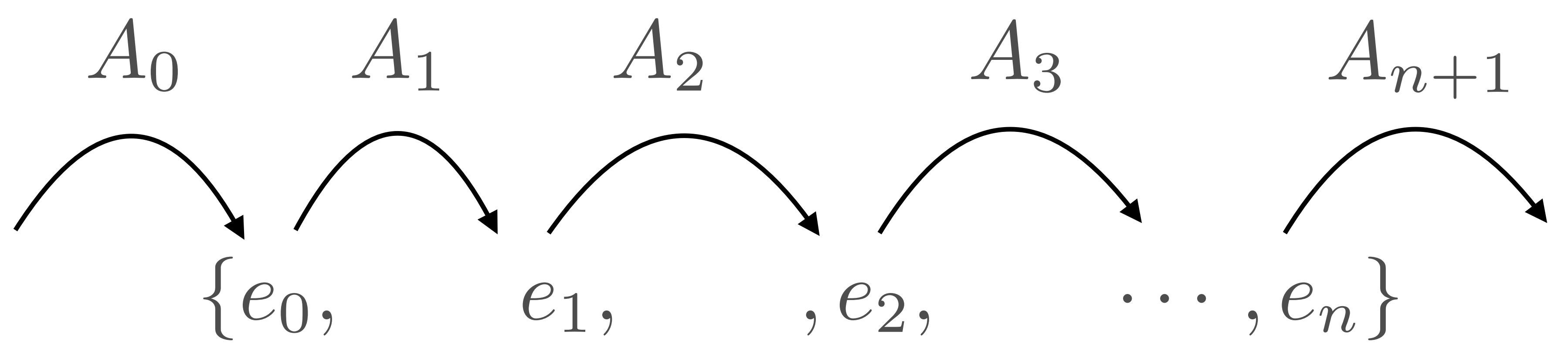

- We want a fold-like way to iterate over the elements of the set (e.g., grammar's rules).
- (Making the set look like a list.)
- But for soundness, the result cannot depend on the order in which the elements are consumed!

Iterating for Follow Calculation

Complexities:

- Processing one sentential form updates *follow* information for multiple non-terminals at once
  - For example, $N \rightarrow aMbPcQ$ gives partial info for follow sets of M, P and Q
- Recursive calls (to N above, say), fold in yet more partial info

The Challenge

✷✷✷ WANTED ✷✷✷

Clean Solutions to
Annoying Problems

But Which Still Refine
To
Efficient Code

The Challenge

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- Iterate ’til convergence (with non-commutative accumulation) *is* possible
- Preserves grammars as finite sets
- Mostly aesthetically pleasing
- How to then memoize and recombine 3 separate functions?

The Challenge

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Clean Solutions to
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- Iterate 'til convergence (with non-commutative accumulation) *is* possible
- Preserves grammars as finite sets
- Mostly aesthetically pleasing
- How to then memoize and recombine 3 separate functions?
- Translation to CakeML will be easy, (and will re-introduce lists…)

Conclusion

Compilers can be made super formal:

- Programming language semantics and interactive theorem-proving combine to create verified compilers (not only CakeML);
- Compilers use a great deal of theory from many different areas to implement their algorithms;
- Even *first* and *follow* set computations present some interesting challenges…