# COMP3710 (Class \# 5176) <br> Special Topics in Computer Science Computer Microarchitecture 

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## Plan \& Progress

Week 3: Pipelined In-Order (IO) processor
Week 3: Add forwarding to avoid data hazards
This Week: Add hazard detection to avoid load-use hazards
This Week: Mitigate branch hazards + branch prediction

## Big Picture



## Load-Use Data Hazard

| Time (in clock cycles) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CC 1 | CC 2 | CC 3 | CC 4 | CC 5 | CC 6 | CC 7 | CC 8 | CC 9 |

Program execution order
(in instructions

Iw \$2, 20(\$1)
and \$4, \$2, \$5
or \$8, \$2, \$6
add \$9, \$4, \$2
slt \$1, \$6, \$7

Even with forwarding, we need a stall of one cycle

## Detecting Load-Use Hazards

The instruction in the EX And its destination register = stage is a load
if (ID/EX.MemRead and ((ID/EX.RegisterRt = IF/ID.RegisterRs) or (ID/EX.RegisterRt = IF/ID.RegisterRt)) ) stall the pipeline

Perform this check when the "using" instruction is in the decode stage of the pipeline

```
Iw $2, 20($1)\longrightarrowID/EX
    and $4,$2,$5\longrightarrow IF/ID
```


## Stalling the Pipeline

## Front-end:

1. Prevent the IF/ID pipeline register from changing
2. Prevent the PC register from changing


## Stalling the Pipeline

## Front-end:

1. Prevent the IF/ID pipeline register from changing
2. Prevent the PC register from changing

## cycle \# 3



Back-end: Send nops
Executing instruction that have no effect (make all control lines 0)

```
and $4, $2, $5 \longrightarrow IF/ID
or $8,$2,$6\longrightarrowPC
```


## Stalling the Pipeline

## Back-end:

Deassert all control signals in the EX, MEM, and WB stages
(Equivalent to inserting a nop)


Question: Do we need to deassert all control signals or deasserting some of them is sufficient?

## Stalling the Pipeline

## Back-end:

Deassert all control signals in the EX, MEM, and WB stages (Equivalent to inserting a nop)


Question: Do we need to deassert all control signals or deasserting some of them is sufficient?

RegWrite, MemRead, and
MemWrite

## Example: Inserting a Stall

| Time (in clock cycles) |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| CC 1 | CC 2 | CC 3 | CC 4 | CC 5 | CC 6 | CC 7 | CC 8 | CC 9 | CC 10 |

Program
execution
order


## Hazard Detection Unit



## Datapath with Hazard Det.



## Stalls and Performance

Compiler relies on hardware for correct execution
Compiler should still be aware of the microarchitecture for optimally organizaing/reordering the instruction sequence

## Control Hazards

Consider the following instruction sequence

| add | $\$ 4$ | $\$ 5$ | $\$ 6$ |  |
| ---: | :--- | :--- | :--- | :--- |
|  | beq | $\$ 1$ | $\$ 2$ | 40 |
|  | lw | $\$ 3$ | $300(\$ 0)$ |  |
| 40 | or | $\$ 7$ | $\$ 8$ | $\$ 9$ |



1. $P C+4$
2. 40

## Control Hazards

Problem: The pipeline cannot determine the next instruction to fetch, since it only just received the branch instruction from memory (residing in the IF/ID PPR)

Control or branch hazard: When the proper instruction cannot execute in the proper pipeline cycle because the instruction that was fetched is not the one that is needed

1. Stall the pipeline until branch outcome is ready (high penalty)
2. Predict outcome and flush instructions if wrong
3. Seek compiler support (branch delay slot)

## Execution with Taken and Not Taken

cycle \#

cycle \#
Assuming we add extra hardware support to compute the branch outcome in the ID stage, we still incur a 1-cycle penatly when the branch is taken


## Exercise: Performance Penalty

In the SPECint2006 benchmark suite, branches are 17\% (on average) of the total instructions. Assuming branch outcomes are computed in the decode stage, and we stall before fetching the next instruction, what the performance impact of branches on the average CPI. (Assume non-branch instructions have a CPI of 1.)

Answer: 17\%
In deeper pipelines, branches are not resolved much later, and the perforamnce penalty from stalling is much higher

## Control Hazards

Understanding the cost of branch not taken in the absence of hardware support for early branch resolution

Adding extra hardware for early branch resolution

## Control Hazards

|  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Time (in clock cycles) |  |  |  |  |  |  |  |  |  |
|  | CC 1 | CC 2 | CC 3 | CC 4 | CC 5 | CC 6 | CC 7 | CC 8 | CC 9 |

Program
execution
order


## Branch Not Taken

Outcome matches prediction: Noting to do
Outcome does not match prediction:

1. Discard instructions (change the control values to 0)
2. Flush the pipeline (EX, ID, IF)
3. Update the PC with the correct target address

This policy is called static branch prediction: Used in Intel 1486

## Extra Hardware for Taken Branches

How can we reduce the cost of the taken branch?
$\rightarrow$ Branch resolution in MEM stage leads to 3 insts of wasted work
$\rightarrow$ What if we resolve the ID stage? (1 wasted instruction)
Moving the branch in Decode requires two early actions:
$\rightarrow$ Branch decision
$\rightarrow$ Branch target (easy, PC + constant field from IF/ID)
Branch decision
$\rightarrow$ Equality testing (easy, XOR all bits, and then OR then together)
$\rightarrow$ The difficult part is new data hazards in the Decode stage

## Branch Decision in ID Stage

Branch decision
$\rightarrow$ Another forwarding unit in ID stage (formerly only in EX stage)
$\rightarrow$ Hazard detection
Flushing the instruction
$\rightarrow$ Add an extra control line (IF.Flush) that zeros IF/ID PPR (transform to nop)

## Exercise

What happens when the branch is taken in the instruction sequence, assuming the pipeline is optimized for branches that are not taken and that we moved the branch execution to the ID stage?

| 36 | sub | $\$ 10$ | $\$ 4$ | $\$ 8$ |
| :--- | :--- | :--- | :--- | :--- |
| 40 | beq | $\$ 1$ | $\$ 3$ | 7 |
| 44 | and | $\$ 12$ | $\$ 2$ | $\$ 5$ |
| 48 | target $=40+4+7^{\star} 4=72$ |  |  |  |
| 48 | or | $\$ 13$ | $\$ 2$ | $\$ 6$ |
| 52 | add | $\$ 14$ | $\$ 4$ | $\$ 2$ |
| 56 | slt | $\$ 15$ | $\$ 6$ | $\$ 7$ |
| $\cdots$ |  |  |  |  |
| 72 | Iw | $\$ 4$ | $50(\$ 70$ |  |

## Cycle 3



## Cycle 4



## Data Hazards for Branches



How many stall cycles? 0

## Data Hazards for Branches



How many stall cycles? 1

If a comparison register is a destination of preceding ALU instruction or $2^{\text {nd }}$ preceding load instruction

## Data Hazards for Branches

1w \$1, addr<br>beq stalled<br>beq stalled

beq \$1, \$0, target


## Dynamic Branch Prediction



What is the similarity in these pics?
What is different about them?
Imagine your co-pilot has poor navigation skills. What will you do to go at full-speed? What if you drive every day and the decision to go right/left changes over time?

## Fork Context

| Cherry tree: take a left |
| :--- | :--- |
| name:most-recent-turn $\rightarrow$ this-turn |



Wind mills: take a right
Wind mills,
Retzstadt, Franconia, Germany

## Dynamic Branch Prediction

Predict the outcome of a branch instruction (in fetch stage) based on the recent behavior of the branch

## What do we need?

1. Branch identification (name/id, uarch-level)
2. Branch behavior (taken/untaken last time)

## Branch Identification \& Behavior

At the microarchitecture-level, what is the simplest way to name a previously encountered branch
$\rightarrow$ The address of the branch instruction in memory
$\rightarrow$ Instructions are laid out next to each other 4-bytes apart

What is the simplest way to record branch behavior
$\rightarrow$ Outcome (grab it from the ALU)

## Branch History Table

Branch History Table (BHT) or Branch Prediction Buffer
$\rightarrow$ A small amount of memory indexed by the low-order bits of branch address
$\rightarrow$ Keep a single bit that says branch was recently taken or not

$2^{m}$ entries $\rightarrow$ aliasing

## Operation

Placement \& Access: Fetch stage
Predicted untaken: Fetch PC + 4
Predicted taken: Predict/compute target address and fetch target
Outcome matches prediction: Noting to do
Outcome does not match prediction:

1. Flip the entry in the BHT
2. Flush the pipeline (EX, ID, IF), update PC

Is correctness affected by misprediction?
Is performance affected by misprediction?

## Accuracy/Perf of 1-bit Predictor

Consider the following loop:

|  | i = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ```for (i=1; i < n; i++) { do something; }``` |  |  |  | T | T | T | T | T | T | T | NT |
|  |  |  |  | T |  | T | T |  | T | T | T |
|  |  |  |  | T | T | T | T | T | T |  | NT |

What is the prediction accuracy of a 1-bit branch predictor if n is equal to 10 ?
$\rightarrow$ Accuracy is $80 \%$ for a branch that is taken $90 \%$ of the time
This example is the reason why forward untaken/backwards taken was used as backup in Intel Pentium Pro, II, III, 4

## Anomalous Decision

When branches that are strongly biased toward one direction suddenly takes a different path/direction
E.g., Branches at the end of a loop are usually taken (backward) except for the case of the loop exit (forward)

A 1-bit predictor is "thrown off" by a single anomolous decision

## Smith's Algorithm

1979: James E. Smith patents branch prediction at Control Data


Notices the performance pathology of 1-bit predictor at loop termination Key insight: Add hysterisis (inertia) to the predictor's state


## $\mathrm{k}=2$

A saturating counter maps the outcomes of several recent branches on to a counter with different states

Four states:

1. Strongly not-taken (SN or 00)
2. Weakly not-taken (WN or 01)
3. Weakly taken (WT or 10)
4. Strongly taken (ST or 11)

The same outcome must occur multiple times to reach the strong states

## State Diagram with k=2

The state transitions show the bimodal behavior of Smith predictor


## Accuracy of Smith's Predictor

Accuracy of Smith (1-bit counter) and Smith (2-bit counter) on a sequence of branches with a single anomolous decision

| Branch | Branch <br> Direction | Smith $_{1}(\mathrm{k}=1)$ | Smith $_{2}(\mathrm{k}=2)$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | 1 | State | Prediction | State | Prediction |
| B | 1 | 1 | 1 | 11 | 1 |
| C | 0 <br> anomaly | 1 | 1 <br> (misprediction) | 11 | 1 <br> (misprediction) $)$ |
| D | 1 <br> anomaly | 0 | 0 <br> (misprediction) | 10 | 1 |
| E | 1 | 1 | 1 | 11 | 1 |
| F | 1 | 1 | 1 | 11 | 1 |

## END: Week 4, Part 1

What is interesting about $\mathrm{B} 1, \mathrm{~B} 2$, and B 3 ?


## Branch Correlation

What is interesting about $\mathrm{B} 1, \mathrm{~B} 2$, and B 3 ?


## Branch Correlation

Insight \# 1, Global Branch Correlation: Behavior of one branch is often correlated with the behavior of other branches

Insight \# 2, Local Branch Correlation: Behavior of a branch is often correlated with the past outcomes of the same branch other than the outcome of the branch the last time it was executed

```
for (i= 1; i < n; i++)
{
    do something;
}
n=3:11011011011011011011
n=4:11101110111011101110
```

Capturing four recent branch outcomes in a window reveal distinct patterns
$\mathrm{n}=3$ : 11011011011011011011
$\mathrm{n}=4: 11101110111011101110$

## Correlating Branch Predictors

Branch predictors that use the behavior (outcomes) of other branches to make a prediction

A predictor that uses the outcomes of only a single branch to predict the behavior of that branch does not capture correlation b/w branches

Also called two-level adaptive branch prediction
$\rightarrow$ Two separate levels of branch history to make the prediction

## Recording Branch History

The global history of the most recent $m$ branches can be recorded in an m-bit shift register (branch history register or BHR)

| $$ | 2-bit shift register |  |  |
| :---: | :---: | :---: | :---: |
|  | 0 | 0 | initial state |
| \% | 0 | 1 | B1 taken |
|  | 1 | 1 | B2 taken |
| $\pm$ | 1 | 0 | B3 not taken |

3-bit shift register

| 0 | 0 | 0 | initial state |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 1 | B1 taken |
| 0 | 1 | 1 | B2 taken |
| 1 | 1 | 0 | B3 not taken |

Shift the actual outcome at one end and discard the oldest outcome

## Global-History Two-Level Predictor

Pattern History Table (PHT): A table of saturating 2-bit counters. Indexed with a concatenation of BHR and a selection of branch address bits


## Operation of gselect

The counters in the indexed PHT provides prediction (and are updated) in a manner similar to Smith's algorithm

The contents of BHR are shifted with the correct outcome of the branch

PHT is indexed with a concatenation of address and history bits


## Intuition for gselect

Behavior of one branch is correlated with that of an earlier branch

1. The branches test conditions involving the same variable
2. One instruction gaurds a variable that another one tests

## Irrelevant Branches \& Training Time

```
x = 0
if (something) /* branch A */
    x = 3;
if (someothercondition) /* branch B */
    y += 19;
if (x <= 0) /* branch C */
    dosomething();
```

| C | B | A |
| :---: | :---: | :---: |
| 1 | 0 | 0 |
| 0 | 0 | 1 |
| 1 | 1 | 0 |
| 0 | 1 | 1 |

1. Tracking A could help achieve perfect prediction for $C$
2. The "irrelevant" branch B increases the training time of gselect (predictor must learn to ignore these irrelevant history bits)

## Size of PHT

For a harware budget of 4 KB , how many entries can reside in the PHT at once? (Assume 2-bit saturating counters)

General formula: For an X KB PHT, the PHT contains $4 * X$ 2-bit counters (Each byte can hold 4 2-bit saturating counters)

## Indexing the PHT

History bits: h
Address bits: m
\# entries in the PHT: $2^{\text {h+m }}$
For a 4 KB PHT with 16,384 entries, we have 14 bits (total)
Distributing 14 bits across history and address exposes interesting tradeoffs

## PHT Indexing: \# Address Bits Matter



Any branch that ends in 01 share the same entries in the PHT
$\rightarrow$ Navigation Analogy: Morrison Street, Morrison Road, Morrison Boulevard

1 Branch Prediction

## PHT Indexing: \# History Bits Matter

Using more history bits allows the predictor to correlate against more complex branch history patterns

Optimally balancing the address vs. history bits depends on: (1) compiler's arrangement of the code, (2) program, (3) ISA, and (4) program inputs (anything else?)

## Local-History Two-Level Predictor

Global History: Track outcomes of several branches encountered
Local History: Track the outcomes of the last several outcomes of only the current branch

Navigation Analogy

$\rightarrow$ Intersection R, weekdays right (work ${ }^{-\theta}$ ), weekends left (downtown )
$\rightarrow$ Turns made to get to the intersection (i.e., global history) are the same
$\rightarrow$ If $R, R, R, L$ is the local (intersection $R$ ) history, then what day is next?

Remembering driving decision history for each intersection requires more information than the local history

## Local-History Two-Level Predictor



## Local-History Two-Level Predictor

Replace global BHR with a one BHR per branch
$\rightarrow B H R=B H T$ with one entry
Combine BHR contents with selected PC bits to index the PHT
Smith's saturating counters work as before
Shift the most recent branch outcome into the selected entry into the BHT


## Sizing Tradeoffs

1. Balancing the \# history and \# non-history (address) bits for the PHT index
2. For a given budget, how many bits for the BHT and how many for the PHT
3. BHT: \# entries and the width of each entry (history length)

Sizing Formula: For a predictor with an L-entry BHT and an h-bit history, and one that uses $m$ bits of the branch address for the PHT index requires a total size of $L^{*} h+2^{h+m+1}$

## Working Example



## Outcome = Taken



## Intel P6

Uses Local History 2-level predictor with a history length of 4
... with "slight" complications


## Loop Closing Example

11101110111011101110


## The (m,n) View

So far, we have considered a monolithic view of the PHT An alternative view is the ( $m, n$ ) view shown below There are $2^{m}$ PHTs, and each entry of the PHT is n-bits wide


What is a $(2,2)$ predictor? \# PHTs? Smith $_{k}(\mathrm{k}=$ ? $)$
What is a $(0,2)$ predictor? What about $(0,1)$ ?

## Exercise

How many bits are in (0,2) branch predictor with 4 K entries?

How many branch-selected entries are in a $(2,2)$ predictor that has a total of 8 K bits in the prediction buffer (monolithic PHT)?


## Solution

How many bits are in $(0,2)$ branch predictor with $4 K$ entries?

- A $(0,2)$ branch predictor does not use any history at all since $m=0$. The $(0,2)$ predictor also uses 2-bit saturating counters. It is essentially a Smith ${ }_{2}$ predictor. Since each entry of the branch prediction buffer is 2 bits (counter) and there are 4 K entries, the total number of bits in a $(0,2)$ predictor equals 8 Kbits.


## Solution

How many branch-selected (i.e., $2^{p}$ ) entries are in a $(2,2)$ predictor that has a total of 8 K bits in the prediction buffer (monolithic PHT)?

- A $(2,2)$ predictor uses 2 history bits and therefore the number of PHTs is equal to 4. The size of each PHT is then 2 Kbits. Since each entry of the PHT is 2-bits wide (i.e., $n=$ 2), there are $1 K$ entries in the PHT. Thus, the branch selected entries in the PHT is equal to 1 K .


## Taxonomy of 2-Level Predictors

What type of branch historty to use?

- Global (G)
- Per-address (P)

Where does the non-history bits in the index come from?

- Ignore PC, use BHR only (g) $\rightarrow$ global branch history table (gPHT)
- Use low-order bits of the address (p) $\rightarrow$ per-add branch history table (pPHT)

Four predictors

- GAg
- GAp
- PAg
- PAp


## Four predictors

- GAg
- GAp
- PAg
- PAp



## Index Sharing Predictors

Motivation: A drawback of 2-level algorithms is that the designer must make a tradeoff b/w BHR width and \# branch address bits to index the PHT

Example: Consider the loop closing branch pattern again. The history length is 4 bits. With 4 bits, we can access 16 PHT entries. But there are only four frequently occuring patterns

Index formation in gselect and other 2-level correlating predictors introduces inefficiencies

## McFarling's gshare Predictor

McFarling (1993) proposed a variation of the global-history 2-level predictor

Key Insight: Make better use of index bits by hashing the BHR and PC together to index the PHT

Why does it works? Combining BHR and PC contains more information due to the non-uniform distribution of branch addresses and history bits (index sharing)

## Indexing: GAp vs. gshare

GAp: 4-bit index ( 2 bits from BHR and 2 low order bits from PC)

| BHR | 1 | 1 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- |
| PC | 0 | 1 | 1 | 0 |
| Index | $?$ | $?$ | $?$ | $?$ |


| BHR | 1 | 0 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- |
| PC | 1 | 0 | 1 | 0 |
| Index | $?$ | $?$ | $?$ | $?$ |


| BHR | 1 | 1 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- |
| PC | 0 | 1 | 1 | 0 |
| Index | $?$ | $?$ | $?$ | $?$ |


| BHR | 1 | 0 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- |
| PC | 1 | 0 | 1 | 0 |
| Index | $?$ | $?$ | $?$ | $?$ |

## Indexing: GAp vs. gshare

GAp: 4-bit index (2 bits from BHR and 2 low order bits from PC)

| BHR | 1 | 1 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- |
| PC | 0 | 1 | 1 | 0 |
| Index | 1 | 0 | 0 | 1 |


| BHR | 1 | 0 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- |
| PC | 1 | 0 | 1 | 0 |
| Index | 1 | 0 | 0 | 1 |

Information loss during index formation

| BHR | 1 | 1 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- |
| PC | 0 | 1 | 1 | 0 |
| Index | 1 | 0 | 1 | 1 |


| BHR | 1 | 0 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- |
| PC | 1 | 0 | 1 | 0 |
| Index | 0 | 0 | 1 | 1 |

## gshare Circuit



## pshare Circuit



## More Real-World Examples

IBM Power 4
$\rightarrow$ Global history predictor
$\rightarrow$ 11-bit BHR
$\rightarrow 16,384$ entry PHT
$\rightarrow$ 2nd level uses Smith ${ }_{1}$
Power 4 Core: http://ixbtlabs.com/articles/ibmpower4/


## More Real-World Examples

Alpha 21264
$\rightarrow$ Global history predictor
$\rightarrow$ 12-bit BHR
$\rightarrow 4096$ entry PHT

R. E. Kessler, "The Alpha 21264 Microprocessor," IEEE MICRO

## Questions to Ponder

```
int random = rand() % 100; /* naïve/poor way to get a random no. */
if (some_condition_involving_random)
{
    do something;
}
```

How should we deal with probalistic branches (increasingly common)?
$\rightarrow$ On Alpha 21264, the misprediction penalty is not 1 cycle, but 14 cycles
$\rightarrow$ Is it worth flushing the pipeline for a misprediction on a probablistic branch? What does the program semantics demands? What is programmer's intention?

## Questions to Ponder: Role of ML

- Should we pay attention to machine learning for building more accurate branch predictors? What has changed in recent times?
- Movement away from desktop to the cloud
- Are the workloads in the cloud converging to a few classes?

Is user diversity equal to workload diversity? How can we exploit these trends for optimizing processors?

- Big Profiling Data, Huge and diverse datasets


## Next Two Weeks

Tournament Predictor
Tagged Hybrid Predictor (TAGE)
The Perceptron Predictor (time permiting)
Branch Target Buffer
Exception Handling (MIPS Pipeline)
Week 6: In-Order to Out-of-Order Transformation

## Plan

Week 4: Data and branch hazards, branch prediction
Week 5: Correlating predictors (via an example)
Week 5: Hybrid, Neural, and Tag-based predictors
Week 5: BTBs, Exception handling, Multiscalar Pipelines

## Homework Solution

Dependence (hazard)
Output (WAW)
Anti (WAR)
True/Data (RAW)
Hazards:
Write-After-Write
Write-After-Read


Read-After-Write

- i2 reads after i1 writes
- i5 writes after i3 has read


## Accuracy Comparison

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T} \mathbf{N}$ Loop Pattern

Compare the accuracy of three predictors

|  | Smith $_{1}$ | Smith $_{2}$ | PAg |
| :---: | :---: | :---: | :---: |
| Accuracy | $?$ | $?$ | $?$ |

## Smith ${ }_{1}$

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{N}$ Actual Direction


|  | Smith $_{1}$ | Smith $_{2}$ | PAg |
| :---: | :---: | :---: | :---: |
| Accuracy | $50 \%$ | $?$ | $?$ |

## Smith ${ }_{2}$

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T} \mathbf{N} \mathbf{T}$



|  | Smith $_{1}$ | Smith $_{2}$ | PAg |
| :---: | :---: | :---: | :---: |
| Accuracy | $50 \%$ | $62.5 \%$ | $?$ |

1st iteration: $25 \%$
Steady state: 75\%

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



| PHT |  |
| :---: | :---: |
| 0000 | 00 |
| 0001 | 00 |
| 0010 | 00 |
| 0011 | 00 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | 00 |
| 1100 | 00 |
| 1101 | 00 |
| 1110 | 00 |
| 1111 | 00 |

Iteration \# 1
Prediction: ?
Outcome: ?
Score: ?

## 2-Level PAg predictor

## T T T $\quad$ N T T T T $\mathbf{N} \mathbf{T}$ T T T N T T T



| PHT |  |  |
| :---: | :---: | :---: |
| 0000 | $00 \rightarrow$ | Iteration \# 1 |
| 0001 | 00 | Prediction: NT |
| 0010 | 00 | Outcome: T |
| 0011 | 00 |  |
| 0100 | 00 | Score: 0/1 |
| 0101 | 00 |  |
| 0110 | 00 |  |
| 0111 | 00 |  |
| 1000 | 00 |  |
| 1001 | 00 |  |
| 1010 | 00 |  |
| 1011 | 00 |  |
| 1100 | 00 |  |
| 1101 | 00 |  |
| 1110 | 00 |  |
| 1111 | 00 |  |

## 2-Level PAg predictor

\section*{| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T} \mathbf{T} \mathbf{T} \mathbf{N}$}



|  | PHT |
| :---: | :---: |
| 0000 | $00 \rightarrow$ |
| 0001 | 00 |
| 0010 | 00 |
| 0011 | 00 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | 00 |
| 1100 | 00 |
| 1101 | 00 |
| 1110 | 00 |
| 1111 | 00 |

Iteration \# 1
Prediction: NT
Outcome: T
Score: 0/1

## 2-Level PAg predictor

\section*{| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T} \mathbf{T} \mathbf{T} \mathbf{N}$}



| PHT |  |
| :---: | :---: |
| 0000 | $00 \rightarrow$ |
| 0001 | $00 \rightarrow$ |
| 0010 | 00 |
| 0011 | 00 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | 00 |
| 1100 | 00 |
| 1101 | 00 |
| 1110 | 00 |
| 1111 | 00 |

Iteration \# 1
Prediction: NT
Outcome: T
Score: 0/2

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



|  | PHT |
| :--- | :--- |
| 0000 | $00 \rightarrow 01$ |
| 0001 | $00 \rightarrow 01$ |
| 0010 | 00 |
| 0011 | 00 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | 00 |
| 1100 | 00 |
| 1101 | 00 |
| 1110 | 00 |
| 1111 | 00 |

Iteration \# 1
Prediction: NT
Outcome: T
Score: 0/2

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



|  | PHT |
| :---: | :---: |
| 0000 | 00 |
| 0001 | $00 \rightarrow$ |
| 0010 | 00 |
| 0011 | $00 \rightarrow$ |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | 00 |
| 1100 | 00 |
| 1101 | 00 |
| 1110 | 00 |
| 1111 | 00 |

Iteration \# 1
Prediction: NT
Outcome: T
Score: 0/3

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T} \mathbf{N} \mathbf{T} \mathbf{T} \mathbf{T} \mathbf{N}$



| PHT |  |
| :---: | :---: |
| 0000 | 00 |
| 0001 | 00 |
| 0010 | 00 |
| 0011 | 00 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | 00 |
| 1100 | 00 |
| 1101 | 00 |
| 1110 | 00 |
| 1111 | 00 |

Iteration \# 1
Prediction: NT
Outcome: T
Score: 0/3

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T} \mathbf{N} \mathbf{T} \mathbf{T}$ T



|  | PHT |
| :---: | :---: |
| 0000 | 00 |
| 0001 | $00 \rightarrow$ |
| 0010 | 00 |
| 0011 | $00 \rightarrow$ |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | 00 |
| 1100 | 00 |
| 1101 | 00 |
| 1110 | 00 |
| 1111 | 00 |

Iteration \# 1
Prediction: NT
Outcome: NT
Score: 1/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T} \mathbf{N} \mathbf{T} \mathbf{T}$ T



|  | PHT |
| :---: | :---: |
| 0000 | 00 |
| 0001 | $00 \rightarrow$ |
| 0010 | 00 |
| 0011 | $00 \rightarrow$ |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | 00 |
| 1100 | 00 |
| 1101 | 00 |
| 1110 | 00 |
| 1111 | 00 |

Iteration \# 1
Prediction: NT
Outcome: NT
Score: 1/4

## 2-Level PAg predictor

\section*{| T | T | T | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ |  |  |  |  |  |  |  |  |}



| PHT |  |
| :---: | :---: |
| 0000 | 01 |
| 0001 | 01 |
| 0010 | 00 |
| 0011 | 01 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | 00 |
| 1100 | 00 |
| 1101 | 00 |
| 1110 | 00 |
| 1111 | 00 |

Iteration \# 1
Prediction: NT
Outcome: NT
Score: 1/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



Iteration \# 2
Prediction: NT
Outcome: T
Score: 0/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



Iteration \# 2
Prediction: NT
Outcome: T
Score: 0/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | T | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



|  | PHT |
| :---: | :---: |
| 0000 | 01 |
| 0001 | 01 |
| 0010 | 00 |
| 0011 | 01 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | 00 |
| 1100 | 00 |
| 1101 | $00 \rightarrow$ |
| 1110 | $00 \rightarrow$ |
| 1111 | 00 |

Iteration \# 2
Prediction: NT
Outcome: T
Score: 0/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | T | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



|  | PHT |
| :---: | :---: |
| 0000 | 01 |
| 0001 | 01 |
| 0010 | 00 |
| 0011 | 01 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | 00 |
| 1100 | 00 |
| 1101 | 00 |
| 1110 | $00 \rightarrow$ |
| 1111 | 00 |

Iteration \# 2
Prediction: NT
Outcome: T
Score: 0/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | T | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



|  | PHT |
| :---: | :---: |
| 0000 | 01 |
| 0001 | 01 |
| 0010 | 00 |
| 0011 | 01 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | $00 \rightarrow 01$ |
| 1100 | 00 |
| 1101 | $00 \rightarrow 01$ |
| 1110 | $00 \rightarrow 01$ |
| 1111 | 00 |

Iteration \# 2
Prediction: NT
Outcome: T
Score: 0/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



|  | PHT |
| :---: | :---: |
| 0000 | 01 |
| 0001 | 01 |
| 0010 | 00 |
| 0011 | 01 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | $00 \rightarrow 01$ |
| 1100 | 00 |
| 1101 | $00 \rightarrow 01$ |
| 1110 | $00 \rightarrow 01$ |
| 1111 | 00 |

Iteration \# 2
Prediction: NT
Outcome: T
Score: 0/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



|  | PHT |
| :---: | :---: |
| 0000 | 01 |
| 0001 | 01 |
| 0010 | 00 |
| 0011 | 01 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | $00 \rightarrow 01$ |
| 1100 | 00 |
| 1101 | $00 \rightarrow 01$ |
| 1110 | $00 \rightarrow 01$ |
| 1111 | 00 |

Iteration \# 2
Prediction: NT
Outcome: NT
Score: 1/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



|  | PHT |
| :---: | :---: |
| 0000 | 01 |
| 0001 | 01 |
| 0010 | 00 |
| 0011 | 01 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | $00 \rightarrow 01$ |
| 1100 | 00 |
| 1101 | $00 \rightarrow 01$ |
| 1110 | $00 \rightarrow 01$ |
| 1111 | 00 |

Iteration \# 2
Prediction: NT
Outcome: NT
Score: 1/4

## 2-Level PAg predictor

\section*{| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | T | $\mathbf{T}$ | T | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T} \mathbf{T} . \mathbf{N}$}



|  | PHT |
| :---: | :---: |
| 0000 | 01 |
| 0001 | 01 |
| 0010 | 00 |
| 0011 | 01 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | 01 |
| 1100 | 00 |
| 1101 | 01 |
| 1110 | 01 |
| 1111 | 00 |

Iteration \# 2
Prediction: NT
Outcome: NT
Score: 1/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



Iteration \# 3
Prediction: NT
Outcome: T
Score: 0/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



Iteration \# 3
Prediction: NT
Outcome: T
Score: 0/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



| PHT |  |
| :---: | :---: |
| 0000 | 01 |
| 0001 | 01 |
| 0010 | 00 |
| 0011 | 01 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | 01 |
| 1100 | 00 |
| 1101 | $01 \rightarrow$ |
| 1110 | $01 \rightarrow$ |
| 1111 | 00 |

Iteration \# 3
Prediction: NT
Outcome: T
Score: 0/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{N}$



| PHT |  |
| :---: | :---: |
| 0000 | 01 |
| 0001 | 01 |
| 0010 | 00 |
| 0011 | 01 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | 01 |
| 1100 | 00 |
| 1101 | $01 \rightarrow$ |
| 1110 | $01 \rightarrow$ |
| 1111 | 00 |

Iteration \# 3
Prediction: NT
Outcome: T
Score: 0/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | T | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{T}$ | $\mathbf{N}$ |  |  |  |  |  |  |  |  |  |  |  |  |



|  | PHT |
| :---: | :---: |
| 0000 | 01 |
| 0001 | 01 |
| 0010 | 00 |
| 0011 | 01 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | $01 \rightarrow$ |
| 1100 | 00 |
| 1101 | $01 \rightarrow$ |
| 1110 | $01 \rightarrow$ |
| 1111 | 00 |

Iteration \# 3
Prediction: NT
Outcome: T
Score: 0/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | T | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{T}$ | $\mathbf{N}$ |  |  |  |  |  |  |  |  |  |  |  |  |



|  | PHT |
| :---: | :---: |
| 0000 | 01 |
| 0001 | 01 |
| 0010 | 00 |
| 0011 | 01 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | $01 \rightarrow$ |
| 1100 | 00 |
| 1101 | $01 \rightarrow$ |
| 1110 | $01 \rightarrow$ |
| 1111 | 00 |

Iteration \# 3
Prediction: NT
Outcome: T
Score: 0/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



Iteration \# 3
Prediction: NT
Outcome: NT
Score: 1/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$ T



Iteration \# 3
Prediction: NT
Outcome: NT
Score: 1/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | T | T | T | $\mathbf{N}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$ T



| PHT |  |
| :---: | :---: |
| 0000 | 01 |
| 0001 | 01 |
| 0010 | 00 |
| 0011 | 01 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | 10 |
| 1100 | 00 |
| 1101 | 10 |
| 1110 | 10 |
| 1111 | 00 |

## Iteration \# 3

Prediction: NT
Outcome: NT
Score: 1/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



Iteration \# 4
Prediction: T Outcome: T
Score: 1/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



Iteration \# 4
Prediction: T Outcome: T
Score: 1/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{T}$ | $\mathbf{N}$ |  |  |  |  |  |  |  |  |  |  |  |  |



Iteration \# 4
Prediction: T Outcome: T Score: 2/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{T}$ | $\mathbf{N}$ |  |  |  |  |  |  |  |  |  |  |  |  |



Iteration \# 4
Prediction: T Outcome: T Score: 2/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T} . \mathrm{T} . \mathbf{N}$



Iteration \# 4
Prediction: T Outcome: T Score: 3/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T} . \mathbf{N}$



Iteration \# 4
Prediction: T Outcome: T Score: 3/4

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T}$



Iteration \# 4
Prediction: NT
Outcome: NT
Score: 4/4

100\% accuracy!

## 2-Level PAg predictor

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | T | T |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T} . \mathrm{N}$



|  | PHT |
| :---: | :---: |
| 0000 | 01 |
| 0001 | 01 |
| 0010 | 00 |
| 0011 | 01 |
| 0100 | 00 |
| 0101 | 00 |
| 0110 | 00 |
| 0111 | 00 |
| 1000 | 00 |
| 1001 | 00 |
| 1010 | 00 |
| 1011 | 11 |
| 1100 | 00 |
| 1101 | 11 |
| 1110 | 11 |
| 1111 | 00 |

Iteration \# 4
Prediction: NT
Outcome: NT
Score: 4/4

100\% accuracy!

## Accuracy Comparison

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T} \mathbf{N}$ Loop Pattern

Compare the accuracy of three predictors (4 iterations)

|  | Smith $_{1}$ | Smith $_{2}$ | PAg |
| :---: | :---: | :---: | :---: |
| Accuracy | $50 \%$ | $62.5 \%$ | $43.75 \%$ |

## Accuracy Comparison

| $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{N}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\mathbf{T} \mathbf{N}$ Loop Pattern

Compare the accuracy of three predictors (100 iterations)

|  | Smith $_{1}$ | Smith $_{2}$ | PAg |
| :---: | :---: | :---: | :---: |
| Accuracy | $50 \%$ | $74.5 \%$ | $97.75 \%$ |

## Reasons for Mispredictions

Unseen (cold) branches $\rightarrow$ training time

- Relearning due to phase behavior
- $2^{n}$ history patterns for a BHR of size n

Randomized/cryptographic algorithms
Lack of information

- Global history cannot see local correlation
- Not enough history bits

Negative interference/aliasing

- Contrast with neutral interference


## Types of Aliasing

The three-C's model: Compulsory Capacity Conflict
Compulsory aliasing

- First use of address-history pair (approx. 1\% mispredictions)

Capacity aliasing

- Size of working set is greater than the size of PHT
- 128 -entry PHT, 129 branches in the program

Conflict aliasing

- Two different address-history pairs map to the same PHT entry
- 128-entry PHT, 2 branches in the program with addresses 131 and 259


## Interference-Reducing Predictors

The next two predictors try to reduce the negative interference due to a shared PHT

## gskewed Predictor

Bi-Mode Predictor
Branch Filtering

## gskewed

Operation

- Divide the PHT into multiple banks
- Each bank is indexed with a different hash
- Combine the results with a majority function
- Total update: update all PHTs with the correct outcome
- Partial update: Do not update the mispredicted bank if overall prediction is correct

Intuition: If two branch-history pairs conflict in one PHT, then they are unlikely to conflict in the other two PHTs

## Is gshare not sufficient?

Consider the following branch-history pairs:

| BHR | 0 | 1 | 1 | 0 |
| :--- | :--- | :--- | :--- | :--- |
| PC | 1 | 1 | 0 | 0 |
| Index | 1 | 0 | 1 | 0 |


| BHR | 1 | 1 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- |
| PC | 0 | 1 | 1 | 1 |
| Index | 1 | 0 | 1 | 0 |

## gskewed Predictor



## gskewed Predictor

$$
\begin{aligned}
& f_{0}(x, y)=H(y) \underline{v} H^{-1}(x) \underline{v} x \\
& f_{1}(x, y)=H(y) \underline{v} H^{-1}(x) \underline{v} y \\
& f_{2}(x, y)=H^{-1}(y) \underline{\vee} H(x) \underline{v} x
\end{aligned}
$$

- ( $x, y$ ) are $n$-bits long
- $\mathrm{H}^{-1}$ is inverse of H

$$
f_{0}(x 1)=f_{0}(x 2) \text { then } f_{1}(x 1)!=f_{1}(x 2) \text { and } f_{2}(x 1)!=f_{2}(x 2)
$$

$H\left(b_{n}, b_{n-1}, \ldots, b_{3}, b_{2}, b_{1}\right)=\left(b_{n}\right.$ XOR $\left.b_{1}, b_{n}, b_{n-1}, \ldots, b_{3}, b_{2}\right)$

## gskewed Predictor

Used in Alpha EV8 Never realized


## Bi-Mode Predictor

Operation

- Split branches into two groups (ST and SNT)
- Use two PHTs (direction predictors) and index with the same address-history hash
- ST branches map to one PHT, and SNT branches to the other
- A meta-predictor (choice predictor) selects the PHT bank
- Index the choice predictor with the branch address

Intuition: Branches have a bias (ST or SNT). Separating them into two PHT mitigates -ve interference. If two branches map to the same entry in the choice predictor, they are unlikely to harm each other

## Bi-Mode Predictor



## Bi-Mode Predictor

15\% of Cortex A15 power

Sizing more complicated as one needs to tune PHT sizes and that of the choice predictor


## Branch Filtering

Intuition: Reduce the \# branches stored in the PHT by removing highly biaseed branches from the PHT

Operation

- Track how many times a branch has gone in the same direction
- Beyond a threshold, a branch is "filtered" and no longer updates the PHT
- If the direction changes, reset the counter, and note the new direction


## Branch Filtering

## gshare



## Alternative Context Predictors

Tradeoffs in choosing the branch prediction context

- Local or global history
- Length of branch history register
- How many bits of the branch address?

Motivation: Can we combine all of the above into a single context? Can we use per-branch-type information? Can we use additional information to form context?

## Alloyed History Predictor

Some mispredictions are due to

- Wrong type of history (wrong-history misprediction)
- Some branches prefer local, some global, and some both

Motivation: Distinguish the local and global correlations with the same structure

## Alloyed History Predictor



## Loop Counting Predictors

If we want to accurately predict loops, what size BHR do we need for a loop that iterates n times?

PHT size is exponential in the history length

Loop predictor in Pentium M

- \# iterations (limit)
- Current count
- Direction
- Can detect 11101110 and 00010001


## Loop Counting Predictors

The Pentium-M Loop Predictor Table (One entry)


## The Perceptron Predictor

Motivation: Increasing the \# history bits

- Exponentially increases the PHT size
- Many patterns are irrelevant (training noise)

Question: Can we use more history bits without the exponential increase in area?

- Use perceptron for training the branch predictor
- Use branch history as a feature vector (not index)
https://www.youtube.com/watch?v=5gOTPrxKK60\&ab_channel=Udacity

Perceptron
Table of perceptron weights


## Hybrid Branch Predictors

Motivation: Programs contain a mix of branch types. Different branches may be strongly correlated with different types of history (i.e., global vs local)

Hybrid branch predictors employ two or more single-scheme branch prediction algorithms

- Combine multiple predictions to make one final prediction

McFarling (1993) proposed the multi-scheme tournament predictor

## Tournament Predictor



## Tournament Predictor



## Tournament Meta-Predictor

PO/P1 both correct/incorrect: state unchanged


## Fusion-Based Hybrid Predictor

Motivation: Do not throw away the output from any predictor

## Fusion-Based Hybrid Predictor



## TAGE

TAgged GEometric Predictors (state-of-the-art)

Three innovations

- Use multiple history lengths
- History lengths make a geometric series
- Use tags to alleviate aliasing
- Use bits


## Tagged Hybrid Predictors



## Pipeline Tradeoff Analysis Peter Kogge, 1981

$$
\begin{aligned}
& \mathrm{C}=G+k * L \\
& P=\frac{1}{\left(\frac{T}{k}+S\right)}
\end{aligned}
$$

P = Performance
T = non-pipelined-delay
k = depth of the pipeline (\# stages)
$\mathbf{S}=$ Delay due to the addition of a latch (pipeline register)
C = Cost
G = Cost in terms of gates in non-pipelined
L = Cost of adding each latch

We can find $C / P$, the cost to performance ratio, and then take the first derivative, set it to 0 , and solve for

$$
k_{o p t}=\sqrt{\frac{G T}{L S}}
$$ the value of $k$ that minimizes the cost/performance ratio

## Pipeline Tradeoff Analysis



## More Recent Study

A. Harstein and Thomas R. Puzak, "The Optimum Pipeline Depth for a Microprocessor," ISCA, 2002

