COMP4610/COMP6461

Week 5 - Vectors and Matrix Math

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Linear Algebra

[Useful Resources]

- Khan Academy is good
  https://www.khanacademy.org/math/linear-algebra
- Wikipedia is not bad
  https://en.wikipedia.org/wiki/Linear_algebra

Vector Math

- Addition
- Multiply by Scalar
- Element-wise multiplication (Hadamard product)
- Dot Product, and length.
- Cross Product

Coordinate Systems

- Polar Coordinates
- Cylindrical Coordinates
- Spherical Coordinates

3D Transformations

3D Matrix Transformation

Translation and scaling in 3D are very similar to that of 2D.

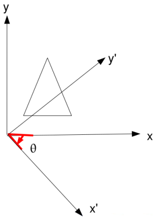
$$T(t_x, t_y, t_z) = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

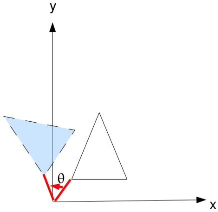
$$S(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2D Rotation

2D rotation can be viewed as a **change of basis**.

$$R_{(x',y')} = \begin{bmatrix} x'_x & x'_y & 0 \\ y'_x & y'_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & -\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$





3D Rotations - Euler Angles

3D rotations can be specified by a sequence of rotations around the axes. (pitch, roll, and yaw)

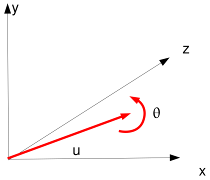
$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note: The inverse of a rotation matrix is it's transpose $R^{-1} = R^T$

3D Rotation - Unit vector + Angle

3D Rotations can be also specified in terms of a unit vector along with an angle of rotation.

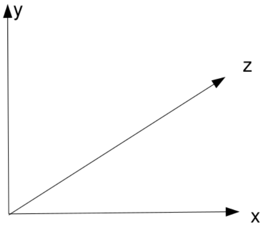


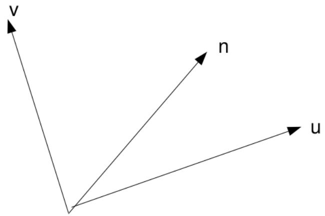
$$R_u(\theta) = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \sin\theta + \mathsf{I}_3 \cos\theta + uu^T(1 - \cos\theta)$$

Where $u = \begin{bmatrix} x & y & z \end{bmatrix}^T$, $|u|_2 = 1$, and I_3 is the 3x3 identity matrix

3D Rotation - Change of basis

We can also think of 3D rotations in terms of a change of basis.





$$R_{uvn} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D Rotation - Summary

Several ways to specify rotation in 3D...

- Euler angles (pitch, roll, yaw)
- Angle around a unit vector
- Specification of new basis vectors
- Quarternians (not covered in this course)