Computational Complexity

C5

Time and Space Complexity

Big O Notation

Examples

Practical Study: Sets

Context

Key computational resources:

- Time
- Space
- Energy

Computational complexity is the study of how problem size affects resource consumption for a given implementation.

- Worst case
  - the complexity of solving the problem for the worst input of size *n*
- Average case
  - is the complexity of solving the problem on an average.

Broad Approach

1. Identify *n*, the number that characterizes the problem size.
   - Number of pixels on screen
   - Number of elements to be sorted
   - etc.
2. Study the algorithm to determine how resource consumption changes as a function of *n.*

Big O Notation

Suppose we have a problem of size n that takes $g(n)$ time to execute in the average case.

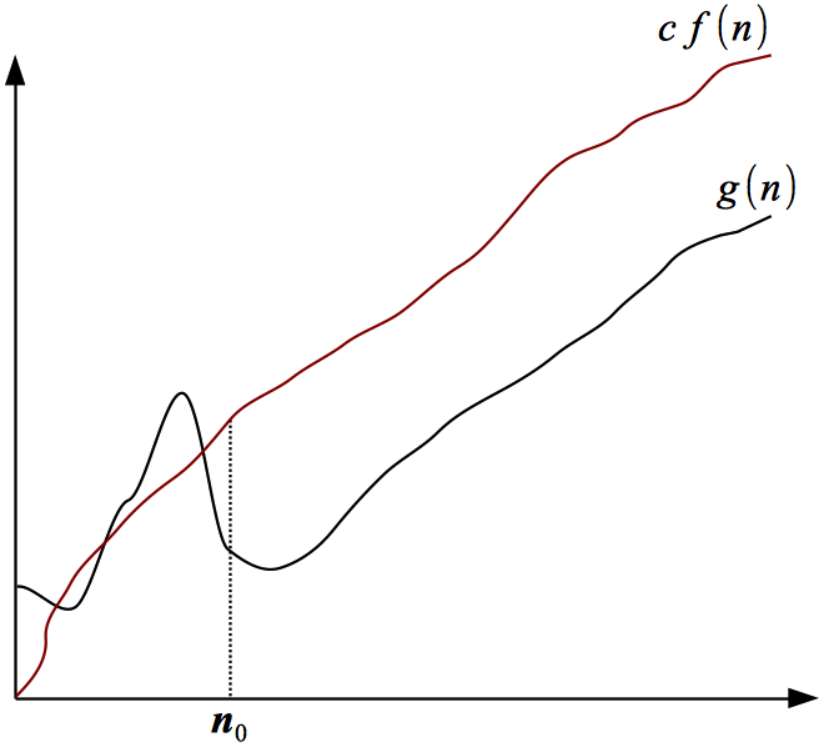
We say:

$g(n) \in O(f(n))$

if and only if there exists a constant $c > 0$

and a constant $n_0 > 0$ such that for all $n > n_0$:

$g(n) \leq c \times f(n)$



Simple Examples

- Constant $O(1)$
  - Time to perform an addition
- Logarithmic $O(\log(n))$
  - Time to find an element in a (balanced) BST
- Linear $O(n)$
  - Time to find an element within a list
- $O(n \log(n))$
  - Average time to sort using mergesort
- Quadratic $O(n^2)$
  - Time to compare n elements with each other

Time Complexity: Counting Statements

Time complexity can estimated by simply counting the number of statements to be executed.

- Traps
  - Simple statements are constant time
  - Library calls may have arbitrary complexity

Concrete Examples

Consider hashing into a table of *n* elements…

```
public int hash(Integer key, int buckets) {
        return key % buckets;
}
```

Constant time, *O*(1)

Concrete Examples

Consider summing a list of size *n*…

```
public int sum(ArrayList<Integer> list) {
  int rtn = 0;
  for(Integer i: list) {
          rtn += i;
  return rtn;
}
```

Linear time, $O(n)$

Concrete Examples

```
public int minDiff(ArrayList<Integer> values) {
 int min = Integer.MAX_VALUE;                          1
 for (int i = 0; i < values.size(); i++) {             n
  for (int j = i + 1; j < values.size(); j++) {        (n – 1)n/2
   int diff = values.get(i)-values.get(j);             (n – 1)n/2
   if (Math.abs(diff) < min)                           (n – 1)n/2
    min = Math.abs(diff);                              (n – 1)n/2
  }
 }
}
```

$$S(N) = 1 + n + 4\,((n-1)\,n/2) = 1 + n + 2\,n^2 - 2n = 2n^2 - n + 1 \in O(n^2)$$

Note: n -1 + n – 2 + … 2 + 1 = (n – 1) n /2