## Game Playing AI

## Guest Lecture in Structured Programming

Pascal Bercher<br>Many thanks to Stephen Gould!<br>Slides partially build upon his lecture from 2019.<br>Planning \& Optimization Group<br>College of Engineering and Computer Science<br>the Australian National University (ANU)<br>August 21, Semester 2, 2020



Australian
National
University

- Motivation: Why Solving Games Automatically Anyways?
- What are Games? (A few Definitions)
- Solving Small Games
- MiniMax
- $\alpha / \beta$ Pruning
- Games with Chance
- Solving Large Games
- Defeating Dragons with Al
- Game AI Success Story
- Game Als for computer games (modern ones or board game adaptations).


## Why Bother? Why Solving Games Automatically?

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- Purely for the sake of knowledge!
E.g., can you always (force a) win in "Connect 4" when you start?



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- Robotics or Multi-Agent-Planning (though this is often cooperative, whereas we take a look at antagonistic games)
- Economics! Cf. game theory (look up: Nash Equilibrium and Prisoner's Dilemma)


## What are Games? Which Kinds Exist?

A game consists of a set of one or more players, a set of moves for the players, and a specification of payoffs (outcomes) for each combination of strategies (also called policy).

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- Zero-sum games vs. non-zero-sum games
- Games with chance (randomness) vs. games without chance



## What's a Strategy?

A strategy defines a complete plan of action for a given player.

Given enough processing time an optimal strategy can be found for games of perfect information by enumerating paths of a game tree. However, in practice this can only be done for small games.


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- Game AI (strategy) vs. game theoretic outcome!

What's the game theoretic outcome?

- The outcome of the game assuming all players play rational.
- Rationality = optimization of expected reward.
- Outcome is known? $\rightarrow$ The respective game is "solved".



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- Just because we have an Al that beats all humans, it doesn't mean the game is solved!

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## MiniMax - How to Solve Small Games?

Using search to solve a game:

- If the game tree is "sufficiently small" we can search in it to find and extract a strategy.
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Consider two players, MAX and MIN. MAX tries to maximize his/her own score, and player Min tries to minimize it.

We assume that the players are rational.

## MiniMax - The MiniMax Algorithm

The MiniMax algorithm allows each player to compute their optimal move on a game tree of alternating MAX and Min nodes.

The value of a node is the payoff for a game that is played optimally from that node until the end of the game.

```
max-value(s)
    if state s is a leaf then
        return payoff(s)
    v:=-\infty
    forall successor states s' of s do
    Lv:= max {v, min-value(s')}
    return v
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## MiniMax - Example: Tic Tac Toe

MaX player plays X, Min plays O. Outcomes (black boxes) are from the perspective of the Max player (i.e., 1 is a win, -1 a loss, 0 a draw).

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What is the space requirement of MiniMax?
- We perform a depth-first search!
- So only the longest path needs to be stored.
$\rightarrow$ Space is in $O(b \cdot d)$ (linear)
- MiniMax suffers from the problem that the number of game states it has to examine is always exponential in the number of moves.
- $\alpha / \beta$ pruning is a method for reducing the number of nodes that need to be evaluated by only considering nodes that may be reached in game play.
- Alpha-beta pruning places bounds on the values appearing anywhere along a path:
- $\alpha$ is the best (highest) value found so far for MAX
- $\beta$ is the best (lowest) value found so far for Min
$\alpha$ and $\beta$ propagate down the game tree. $v$ propagates up the game tree.


## $\alpha / \beta$ Pruning — The MiniMax Algorithm Extended By $\alpha / \beta$ Pruning

Keep in mind:

- $\alpha$ is the best value found so far for MAX, initialize with $-\infty$.
- $\beta$ is the best value found so far for Min, initialize with $\infty$.

```
max-value(s, \alpha, \beta)
    if state s is a leaf then
        return payoff(s)
    v := -\infty
    forall successor states s' of s do
        v:= max {v, min-value (s', \alpha,\beta)}
        if v\geq\beta}\mathrm{ then
            return v
        \alpha:= max {\alpha,v}
    return v
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        v:= min {v, max-value(s',\alpha,\beta)}
        if v}\leq\alpha\mathrm{ then
        return v
        \beta:= min {\beta,v}
    return v
```


## $\alpha / \beta$ Pruning - Idea Behind Pruning: When and Why?



## $\alpha / \beta$ Pruning - Example: Tic Tac Toe

Start with $\alpha=-1$ (rather than $-\infty)$ and $\beta=1$ (rather than $\infty$ )

MaX


## $\alpha / \beta$ Pruning - Example: Tic Tac Toe

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What is the runtime (and space requirements) of $\alpha / \beta$ pruning?

- In the worst case: identical to MiniMax! If nothing can be pruned.
- On average: Complexities omitted. (Due to lack of time.)
- This can happen depending on the order in which edges are traversed/payoffs are discovered.
- In practice, it is very unlikely that no pruning occurs, so always choose $\alpha / \beta$ pruning over MiniMax!


## How to Deal with Randomness?

- A random decision can be regarded as the move of yet another player!
- Certainly that's not another Max player! I.e, the "environment" (the random decision) will not always play in our favor!
- But what is it, then?
- Another Min player? (Too pessimistic...)
- If we want to play rational, we maximize the expectation!

$$
\text { value }(s)=\sum_{\text {successor states } s^{\prime} \text { of } s} P\left(s^{\prime}\right) \cdot \text { value }\left(s^{\prime}\right)
$$

Illustration For a 2-Player Game With Throwing Two Dice, Counting Their Sum


When is using MiniMax and $\alpha / \beta$ Pruning still feasible?

- Recall that the complexity of MiniMax (and $\alpha / \beta$ !) is exponential! I.e., in $O\left(b^{d}\right)$, with
- $b$, the branching factor (available moves per state)
- $d$, the depth (number of moves until game ends)

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- Recall that the complexity of MiniMax (and $\alpha / \beta$ !) is exponential! I.e., in $O\left(b^{d}\right)$, with
- $b$, the branching factor (available moves per state)
- $d$, the depth (number of moves until game ends)
- For some games that is simply too large!
- So, let's take a look at some examples...

Examples for (estimated) number of reachable (game) states: (Source: https://en.wikipedia.org/wiki/Game_complexity)

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- Maximum after duplicating symmetries: 765


## The "Size" of Games: Tic Tac Toe

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- Rough maximum: $3^{9}=19,683$ (including invalid states)
- Actual maximum: 5,478
- Maximum after duplicating symmetries: 765
- There are still 26, 830 possible games!
(For those states with eliminated duplicates.)
What's a "game"?
A path in the MiniMax tree!


## The "Size" of Games: Connect 4

Examples for (estimated) number of reachable (game) states:
(Source: https://en.wikipedia.org/wiki/Connect_Four)

- Rough maximum: $3^{7.6}<1.110^{20}$ (including invalid states)
- Actual maximum: $4,531,985,219,092 \approx 4.5 \cdot 10^{12}$ (still including symmetries)
- First solved, independently, by James Dow Allen (October 1, 1988), and Victor Allis (October 16, 1988).
- Note that today it can also be solved using $\alpha / \beta$ pruning!


## The "Size" of Games: Blokus

Examples for (estimated) number of reachable (game) states:
(Source: by Stephen Gould, previous year(s))

approximatelty 58 moves, not all symmetries eliminated

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## The "Size" of Games: Chess

Examples for (estimated) number of reachable (game) states:
(Source: https://en.wikipedia.org/wiki/Shannon_number)

- Some maximum: $5 \cdot 10^{52}$
- Lower limit on game tree size: $10^{123}$
- More conservative estimate on lower limit of game tree size, eliminating obvious bad moves: $10^{40}$


## The "Size" of Games: Go

Examples for (estimated) number of reachable (game) states:
(Source: https://en.wikipedia.org/wiki/Shannon_number)

- Legal positions: $2.08168199382 \cdot 10^{170}$
- Lower limit on number of games: $10^{\left(10^{48}\right)}$
- Upper limit on number of games: $10^{\left(10^{171}\right)}$


## How to deal with large games?

So, what to do for (too) large games?

- Don't compute the entire game tree!
- Stop at certain nodes and estimate their payoff!


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- Stop at certain nodes and estimate their payoff! But how?
- hand-crafted heuristics


Black to move

Title: Artificial Intelligence: A Modern Approach (3rd Ed.)
Authors: Stuart Russel and Peter Norvig
URL: https://aima.cs.berkeley.edu/
Estimate per piece:

- pawn: 1 pt
- knight/bishop: 3 pts
rook: 5 pts
queen: 9 pts

Estimate: Black: 7 pts versus White: 6 pts
$\rightarrow$ Black leading! (Only very slightly.)

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- learned heuristics

Machine learning techniques are often used to find a good static evaluation function based on a linear combination of features:

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\hat{v}(s)=w_{1} f_{1}(s)+\cdots+w_{n} f_{n}(s)
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Note the similarity to chess!

- $w_{1}=1, f_{1}(s)=$ number of pawns in $s$
$w_{2}=3, f_{2}(s)=$ number of knights/bishops in $s$


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So, what to do for (too) large games?

- Don't compute the entire game tree!
- Stop at certain nodes and estimate their payoff! But how?
- hand-crafted heuristics
- learned heuristics
- simulate a game, use the outcome as estimate

Monte-Carlo Tree Search is a well-known algorithm exploiting this idea. It works in four phases:

- Selection (select a non-terminal leaf based on current strategy)
- Expansion (expand the selected node)
- Simulation (play a random game to the end)
- Backpropagation (use the outcome to update strategy)

Interested? See, e.g.,
https://www. youtube.com/watch?v=UXW2yZndl7U (15:30, lecture by Dr. John Levine from Univ. of Strathclyde)

## When to use heuristics?

- In standard MiniMax or alpha/beta pruning, we make a terminal test to obtain the payoff, or continue expanding. With heuristics, we instead make a cut-off test to check whether we should stop expansion and estimate the payoff of the current node.


## When to use heuristics?

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- What about using a fixed depth as cut-off test? $\rightarrow$ Suffers from the horizon problem:


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White can promote a pawn into a queen on his next move! So the cut-off test should be negative in this state.

Black to move

## The Assignment: Tsuro of the Seas

But let's start with Tsuro, the "underlying game mechanics".


Figure: YouTube video: https://www.youtube.com/watch?v=MGvY3jsLN1। (1:25) Code: M-G-v-Y-3-j-s-L-N-1 (one)-I(capital-i)

## The Assignment: Tsuro of the Seas

Tsuro of the Seas: Ultra-short introduction


Figure: YouTube video: https://www.youtube.com/watch?v=ziQS8rcT5EA (we just take a glance from 5:04 to 5:58) Code: z-i-Q-S-8-r-c-T-5-E-A

Regarding the game rules: Please stick to the ones officially provided by Steve Blackburn!

Mile Stones in AI Game Playing

## 1959 Arthur Samuel develops Checkers playing program



Mile Stones in Al Game Playing

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2016 Google DeepMind's AlphaGo beats Lee Sedol, Korea

2017 AlphaZero learns Go, Chess, and Shogi from scratch (and beats AlphaGo)


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By Stuart Russel and Peter Norvig from their book Artificial Intelligence: A Modern Approach (3rd Ed.)
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