

# Recursion

## Recursive Algorithms

C1



Australian  
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PAUL  
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A vintage-style advertisement for Droste Cocoa. The background is a solid, vibrant red. At the top center is a royal coat of arms. Below it, the word "Droste" is written in a large, elegant, gold-colored script font with a registered trademark symbol. Underneath the brand name, the word "HOLLAND" is printed in a smaller, gold, sans-serif font. The central figure is a woman dressed as a nun in a white habit and a dark brown robe with white cuffs. She is holding a silver tray with both hands. On the tray, there is a box of Droste Cocoa and a white mug. The box is red and white, featuring the brand name and the nun's image. The mug is white with a small illustration of the nun. At the bottom of the advertisement, the word "cocoa" is written in a large, bold, gold, sans-serif font. Below that, the text "NET WT 250g e 8.8 oz." is printed in a smaller, gold, sans-serif font. At the very bottom left, the word "Kosher" is written in a small, gold, sans-serif font, followed by a U-shaped symbol.

**cocoa**

NET WT 250g e 8.8 oz.

Kosher U

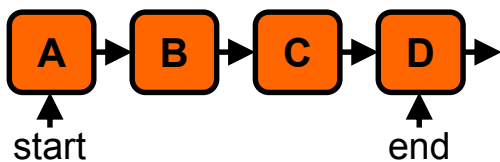


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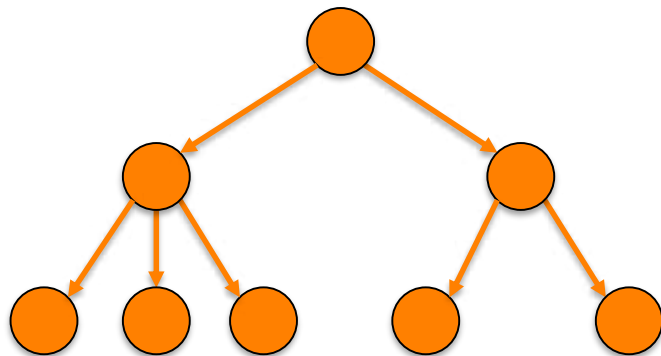


## Recursive Data Structure

A recursive data structure is comprised of components that reference other components of the same type.



linked list



tree

# Recursive Algorithms

A recursive algorithm references itself.

A recursive algorithm is comprised of:

- one or more base cases
- a remainder that reduces to the base case/s

## Example: Fibonacci sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377...

$\text{fib}(0) = 1$  (*base case*)

$\text{fib}(1) = 1$  (*base case*)

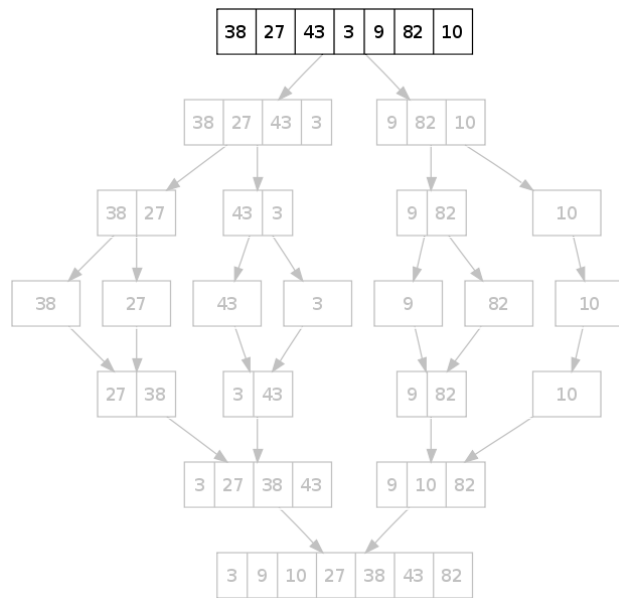
$\text{fib}(n) = \text{fib}(n-1) + \text{fib}(n-2)$  (*for*  $n \geq 2$ )



## Example: Mergesort (von Neumann, 1945)

### Sort a list

- List of size 1 (*base case*)
  - Already sorted
- List of size  $> 1$ 
  - Split into two sub lists
  - Sort each sub list (*recursion*)
  - Merge the two sorted sub lists into one sorted list (by iteratively picking the lower of the two least elements)



Animation: Visualizing Algorithms, Mike Bostock, [bost.ocks.org/mike/algorithms](http://bost.ocks.org/mike/algorithms)



# Hash Functions

# C2

Hash functions  
Choosing a good hash function

# Hash Functions

A hash function is a function  $f(k)$  that maps a key,  $k$ , to a value,  $f(k)$ , within a prescribed range.

A hash is deterministic. (For a given key,  $k$ ,  $f(k)$  will always be the same).

## Choosing a Good Hash Function

A good hash for a given population,  $P$ , of keys,  $k \in P$ , will distribute  $f(k)$  evenly within the prescribed range for the hash.

A *perfect hash* will give a unique  $f(k)$  for each  $k \in P$

# Hashing Applications

# C3

Java hashCode()  
Uses of Hashing

## Java hashCode ()

Java provides a hash code for *every* object

- 32-bit signed integer
- Inherited from `Object`, but may be overridden
- Objects for which `equals ()` is **true** must also have the same `hashCode ()`.
- The hash need not be perfect (i.e. two different objects may share the same hash).

## Uses of Hashing

- Hash table (a map from key to value)
- Pruning a search
  - Looking for duplicates
  - Looking for similar values
- Compression
  - A hash is typically much more compact than the key
- Correctness
  - Checksums can confirm inequality



## Practical Examples...



### Luhn Algorithm

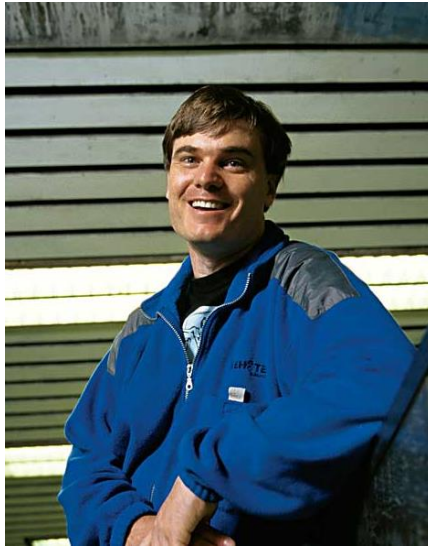
Used to check for transcription errors in credit cards (last digit checksum).



### Hamming Codes

Error correcting codes (as used in EEC memory)

## Practical Examples...



### **rsync (Tridgell)**

Synchronize files by (almost) only moving the parts that are different.



### **MD5 (Rivest)**

Previously used to encode passwords (but no longer).



# Files

# C4

Java File IO

Streams

Standard IO

Random access files

Buffering

# File IO as Streams

A **stream** is a standard abstraction used for files:

- A sequence of values are read.

- A sequence of values are written.

The stream reflects the sequential nature of file IO and the physical characteristics of the media on which files traditionally reside (e.g. tape or a spinning disk).





## Java I/O: Byte Streams

The classes `InputStream` and `OutputStream` allow you to read and write streams of bytes to and from streams including files (subclasses: `FileInputStream` and `FileOutputStream`).

- Open the stream
  - Read or write from the stream (in **bytes**)
  - Wrap operations in a **try** clause
  - Use **finally** to close the streams
- ints** are used, even though **bytes** are transferred(!)

## Java I/O: Character Streams

When reading and writing characters, you should use the classes `Reader` and `Writer`, which allow you to read and write streams of characters to and from streams including files (subclasses: `FileReader` and `FileWriter`).

`ints` are used, even though `chars` are transferred.

## File I/O: Buffering

Reading data one byte at a time is costly. Buffering is used to absorb some of that overhead.

Disk: ~10ms    SSD: ~100 $\mu$ s    RAM: ~10ns    Register: ~1ns

In Java the `BufferedReader` and `BufferedWriter` classes can be used to buffer data read or written with `FileReader` and `FileWriter`.

To be sure that a buffer is flushed, call `flush()`, or close the file.

# Java Command Line IO

Three standard IO streams (globally-defined objects):

- Standard input `System.in`
- Standard output `System.out`
- Standard error `System.err`

```
byte b = (byte) System.in.read();  
System.out.write(b);  
System.out.flush();  
System.err.write(b);
```



## “New” I/O (`java.nio.file`)

Java NIO offers simpler, event-driven interface

- `Path` — replaces `java.io.File`
- `FileSystem` — factory class for objects in the filesystem
- `WatchService` — utility class to detect file system changes through event notification
- `Files` — create, rename, copy, modify attributes and delete files

# Computational Complexity

# C5

Time and Space Complexity

Big O Notation

Examples

Practical Study: Sets

## Context

Key computational resources:

- Time
- Space
- Energy

Computational complexity is the study of how problem size affects resource consumption for a given implementation.

- Worst case
  - the complexity of solving the problem for the worst input of size  $n$
- Average case
  - is the complexity of solving the problem on an average.

## (Computational) Scaling

1. Identify  $n$ , the number that characterizes the problem size.
  - Number of pixels on screen
  - Number of elements to be sorted
  - etc.
2. Study the algorithm to determine how resource consumption changes as a function of  $n$ .

## Big O Notation

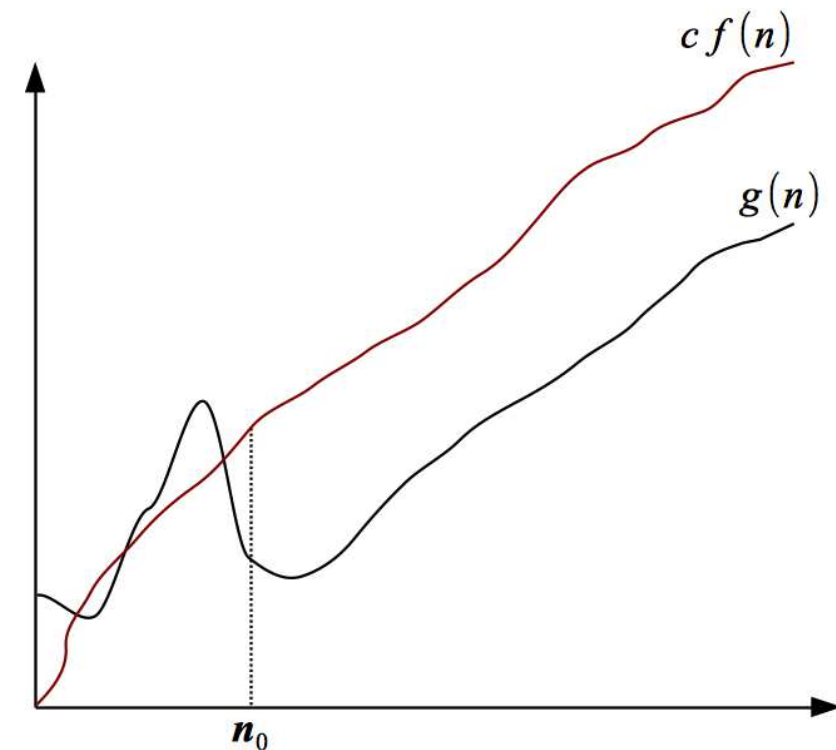
Suppose we have a problem of size  $n$  that takes  $g(n)$  time to execute in the average case.

We say:

$$g(n) \in O(f(n))$$

if and only if there exists a constant  $c > 0$   
and a constant  $n_0 > 0$  such that for all  $n > n_0$ :

$$g(n) \leq c \times f(n)$$



## Simple Examples

- Constant  $O(1)$ 
  - Time to perform an addition
- Logarithmic  $O(\log(n))$ 
  - Time to find an element in a (balanced) BST
- Linear  $O(n)$ 
  - Time to find an element within a list
- $O(n \log(n))$ 
  - Average time to sort using mergesort
- Quadratic  $O(n^2)$ 
  - Time to compare  $n$  elements with each other

## Time Complexity: Counting Statements

Time complexity can be estimated by simply counting the number of statements to be executed.

- Traps
  - Simple statements are constant time
  - Library calls may have arbitrary complexity

## Concrete Examples

Consider hashing into a table of  $n$  elements...

```
public int hash(Integer key, int buckets) {  
    return key % buckets;  
}
```

Constant time,  $O(1)$



## Concrete Examples

Consider summing a list of size  $n$ ...

```
public int sum(ArrayList<Integer> list) {  
    int rtn = 0;  
    for(Integer i: list) {  
        rtn += i;  
    }  
    return rtn;  
}
```

Linear time,  $O(n)$

## Concrete Examples

```

public int minDiff(ArrayList<Integer> values) {
    int min = Integer.MAX_VALUE; 1
    for (int i = 0; i < values.size(); i++) { n
        for (int j = i + 1; j < values.size(); j++) { (n-1)n/2
            int diff = values.get(i) - values.get(j); (n-1)n/2
            if (Math.abs(diff) < min) (n-1)n/2
                min = Math.abs(diff); (n-1)n/2
            }
        }
    }
}

```

$$S(N) = 1 + n + 4 \left( \frac{(n-1)n}{2} \right) = 1 + n + 2n^2 - 2n = 2n^2 - n + 1 \in O(n^2)$$

Note:  $n - 1 + n - 2 + \dots + 2 + 1 = \frac{(n-1)n}{2}$

# Formal Grammars

# C6

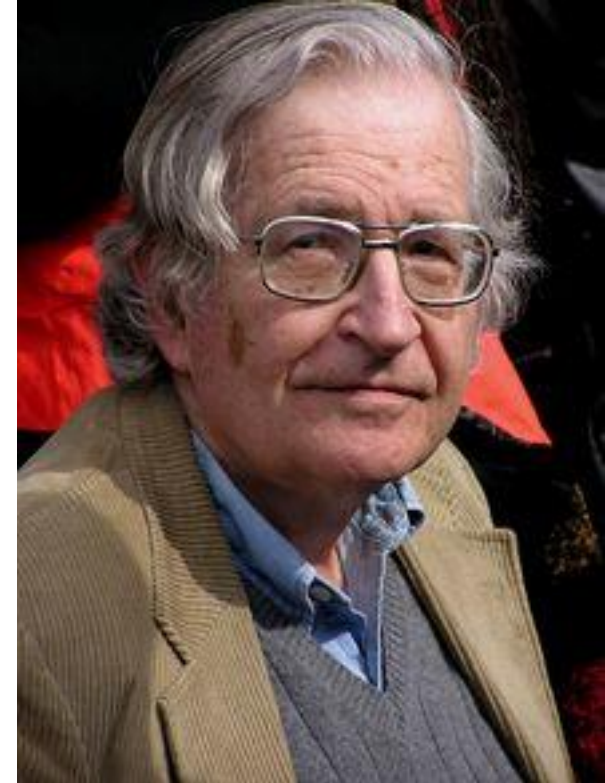
Grammars

EBNF

# Formal Grammars

Formal languages are distinguished from natural languages by their artificial construction (rather than natural emergence).

Noam Chomsky is often credited with opening the field of formal grammars while studying natural languages.



*Duncan Rawlinson (Creative Commons)*

Noam Chomsky

## Generative Grammars

Sentence = Noun Phrase, Verb Phrase, [Noun Phrase];

Noun = signs, directions, lives

Article = the

Verb = show, matter, look

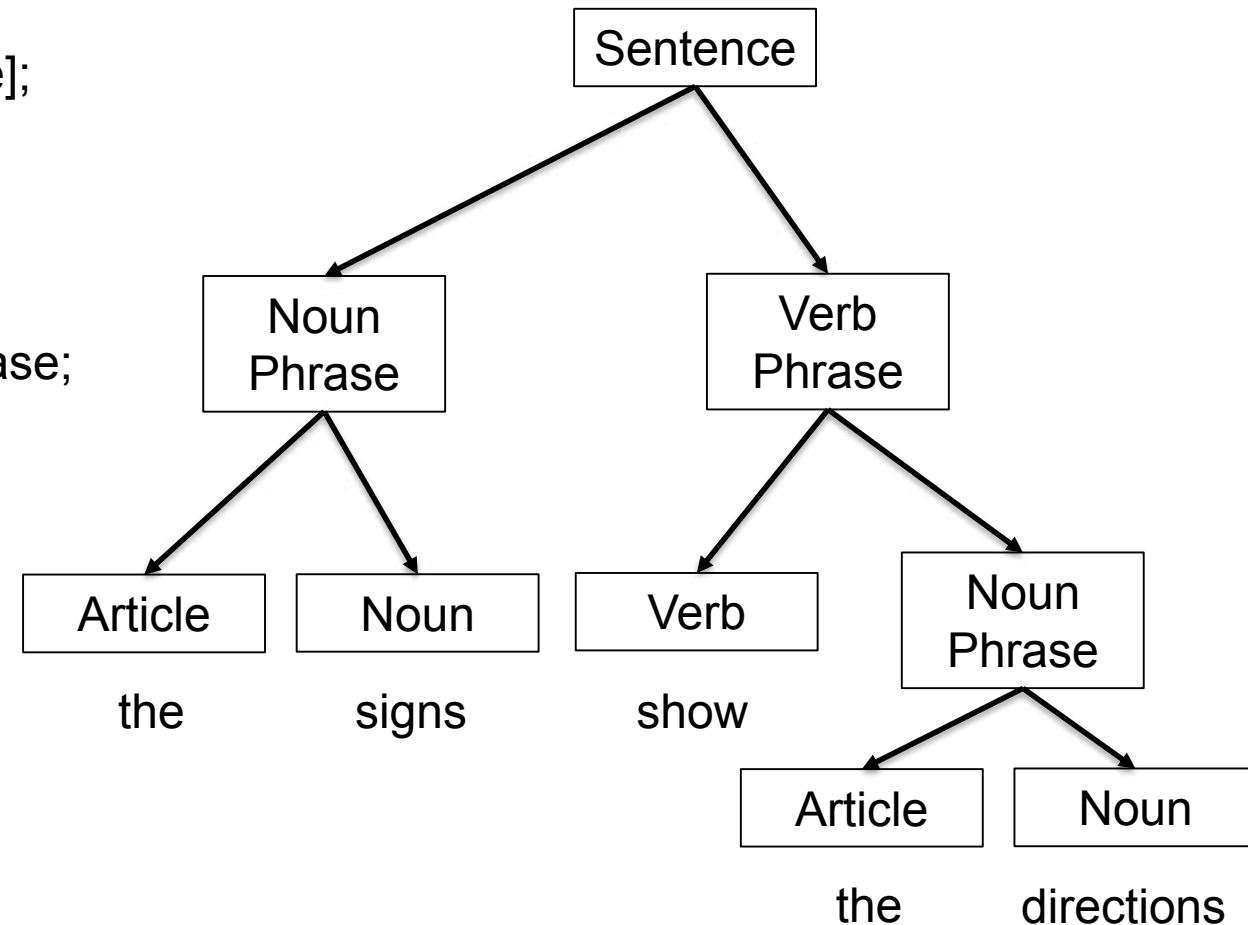
Adjective = big, small, white, black

Noun Phrase = [Article], [Adjective], Noun | Noun Phrase;

Verb Phrase = Verb, [Noun Phrase];

The signs show the directions.

Small big directions matter the black white signs.



# Generative Grammars

Sentence = Noun Phrase, Verb Phrase, [Noun Phrase];

Noun = signs, directions, lives

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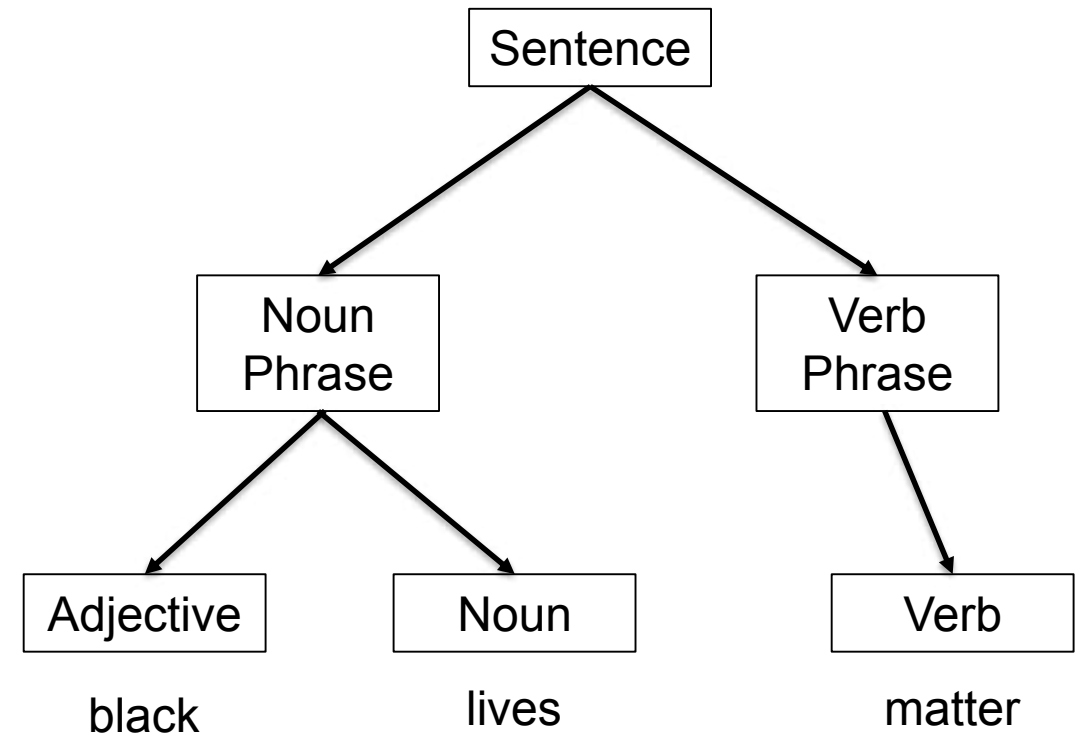
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The signs show the directions.

Small big directions matter the black white signs.



Syntactically correct productions (sentences) don't always convey meaning!

E.g. "I tested positively towards negative."

## Extended Backus-Naur Form

EBNF is a standard way of representing the syntax of a formal language (but *not* the semantics!)

- Terminal symbols
  - e.g. characters or strings
- Production rules
  - combinations of terminal symbols



Robert McClure

Niklaus Wirth

## Extended Backus-Naur Form

Very basic syntax of EBNF production rules:

- '=' defines a production rule
- '|' identifies alternates (e.g. '1' | '2' | '3' )
- '{', '}' identify expressions that may occur zero or more times (e.g. '1', { '0' } )
- '[', ']' identify expressions that may occur zero or one time (e.g. '1', [ '0' ])
- ',' identifies concatenation
- '-' identifies exceptions
- '(', ')' identify groups
- ';' terminates a production rule



## Example EBNF grammar

```
PROGRAM DEMO1
BEGIN
  A0:=3;
  B:=45;
  H:=-100023;
  C:=A;
  D123:=B34A;
  BABOON:=GIRAFFE;
  TEXT:="Hello world!";
END.
```

(\* a simple program syntax in EBNF – Wikipedia \*)

```
program = 'PROGRAM', white space, identifier, white space,
        'BEGIN', white space,
        { assignment, ";", white space },
        'END.' ;

identifier = alphabetic character, { alphabetic character | digit } ;
number = [ "-" ], digit, { digit } ;
string = '"' , { all characters - '"' }, '"' ;
assignment = identifier , "==" , ( number | identifier | string ) ;
alphabetic character = "A" | "B" | "C" | "D" | "E" | "F" | "G"
                    | "H" | "I" | "J" | "K" | "L" | "M" | "N"
                    | "O" | "P" | "Q" | "R" | "S" | "T" | "U"
                    | "V" | "W" | "X" | "Y" | "Z" ;
digit = "0" | "1" | "2" | "3" | "4" | "5" | "6" | "7" | "8" | "9" ;
white space = ? white space characters ? ;
all characters = ? all visible characters ? ;
```

## Simple EBNF Grammars

Grammar for arrangement of characters that are:

- **Natural numbers?**

```
natural = '0' | (nzdigit, { digit } ) ;  
nzdigit = '1' | '2' | '3' | '4' | '5' | '6' | '7' | '8' | '9' ;  
digit = '0' | nzdigit ;
```

- **Integers?**

```
integer = '0' | ([ '-' ], nzdigit, { digit } ) ;
```

- **Decimal numbers?**

```
real = ([ '-' ], natural, [( '.' { digit } , nzdigit )] ) - '-0' ;
```

- **24hr time, digital clock?**

```
time = hour, ':', min ;  
hour = ( ( '0' | '1' ) , digit ) | ( '2' , ( '0' | '1' | '2' | '3' ) ) ;  
min = ( '0' | '1' | '2' | '3' | '4' | '5' ) , digit ;
```

# Threads

## Concurrency

# C7

# Concurrency, Processes and Threads

- Concurrency
  - Multiple activities (*appear to*) occur simultaneously, (e.g. recording this lecture and displaying this slide).
  - ‘Time slicing’ allows a single execution unit to give the appearance of concurrent execution
- Process
  - Distinct execution context that (by default) shares nothing (e.g. IntelliJ, PowerPoint, Quicktime recorder)
- Thread
  - Intra-process execution context (e.g. IntelliJ’s compiler)

# Why Threads?

- ‘Concurrency’
  - Separate concerns (e.g. rendering v logic)
  - Good for: distinct tasks that naturally occur concurrently
- ‘Parallelism’ (a special case of concurrency)
  - Break task into pieces, exploit parallel hardware
  - Good for: computationally intensive problems that can be readily partitioned