

An impressionist painting of a landscape. The scene is dominated by a large, leafy tree in the center, with its branches spreading out. The background shows a row of trees and a building with a red roof. The foreground is a grassy field with yellow flowers. The overall style is characterized by visible brushstrokes and a vibrant, somewhat muted color palette.

A01 Abstract Data Types: Lists

ADTs

List as an ADT

A List interface

Abstract Data Types (ADTs)

Abstract data types* describe the behaviour (semantics) of a data type without specifying its implementation. An ADT is thus **abstract**, not concrete.

- A **container** is a very general ADT, a holder of objects.
- A **list** is an example of a more specific container ADT.

* Not to be confused with: *Algebraic Data Type*.

The List ADT

The **list** ADT is a container known mathematically as a finite sequence of elements. A list has these fundamental properties:

- duplicates are allowed
- order is preserved

A list may* support operations such as these:

- *create*: construct an empty list
- *add*: add an element to the list
- *is empty*: test whether the list is empty

* The operations a given ADT must support will vary depending on the author / library

Our List Interface

We will explore lists using a simple interface:

```
public interface List<T> {  
    void add(T value);  
    T get(int index);  
    int size();  
    T remove(int index);  
    void reverse();  
}
```

```
void add(T value);
```



```
T get(int index);
```



```
int size();
```



```
T remove(int index);
```



```
void reverse();
```



```
String toString();
```



The background of the slide is a painting of a row of trees in a field. The trees are rendered in various shades of blue, green, and brown, with a prominent tree in the center foreground. The style is impressionistic, with visible brushstrokes and a focus on light and color.

A02 List Implementations

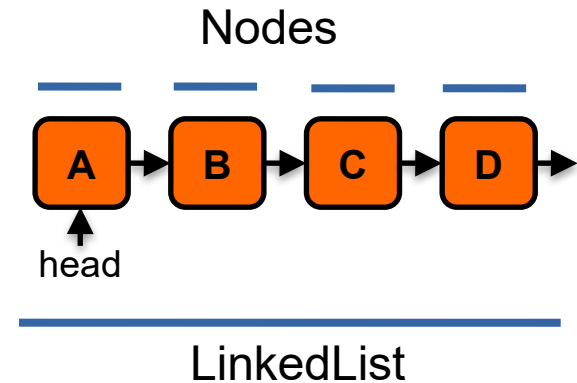
An array-based implementation
A linked-list-based implementation

List Implementation Options

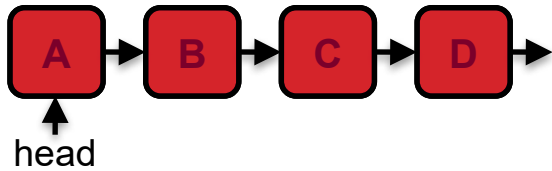
- Arrays
 - Fast lookup of any element
 - A little messy to grow and contract
- Linked list
 - Logical fit to a list, easy to grow, contract
 - Need to traverse list to access elements

Linked Lists: Singly Linked List

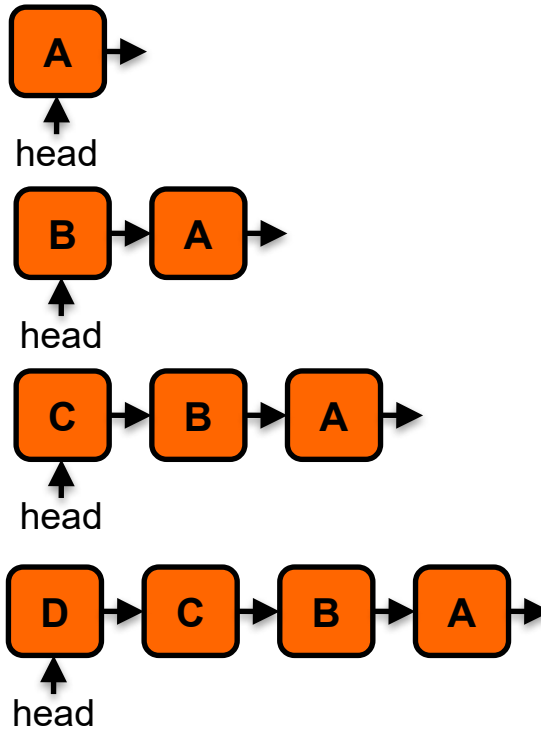
```
public class LinkedList<T> {  
    private class Node<T> {  
        T value;  
        Node<T> next;  
    }  
    Node<T> head;  
}
```



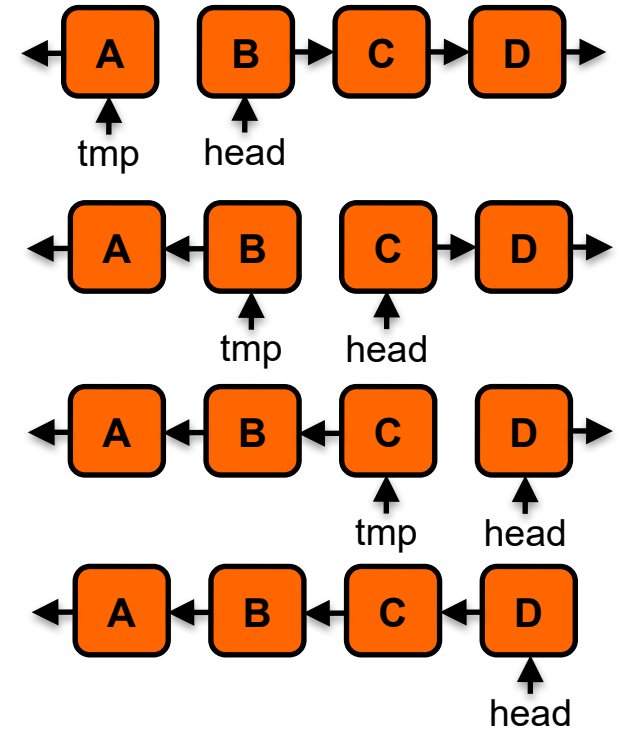
Linked List Reversal: Two Approaches



- Add each item to start:



- Pointer reversal:



Complexity

```
void add(T value);  
T get(int index);  
int size();  
T remove(int index);  
void reverse();
```

ArrayList

- add – Time $O(1)$ amortized, $O(n)$ worst
- get – Time $O(1)$
- size – Time $O(1)$
- remove – Time $O(n)$
- reverse – Time $O(n)$

Space $O(n)$

LinkedList

- add – Time $O(1)$
 - if explicitly tracking last node
- get – Time $O(n)$
- size – Time $O(1)$
 - if explicitly tracked
- remove – Time $O(n)$
- reverse – Time $O(n)$

Space $O(n)$



A03 Sets

Set ADT
A Set interface

The Set ADT

The **set** ADT corresponds to a mathematical set. A set has these fundamental properties:

- duplicates are not allowed
- order is not preserved

A **set** may support operations such as these:

- *create*: construct an empty set
- *add*: add an element to the set
- *contains*: does the set contain a given element
- *remove*: remove an element from the set

Our Set Interface

We will explore sets using a simple interface:

```
public interface Set<T> {  
    boolean add(T value);  
    boolean contains(T value);  
    int size();  
    boolean remove(T value);  
}
```



A04 Sets: HashSet

Hash tables

A hash-table-based Set implementation

Hash Tables

Stores **keys**, using a hash function to map a key into a table entry. Optionally, **values** can be associated with keys and stored alongside them in the table.

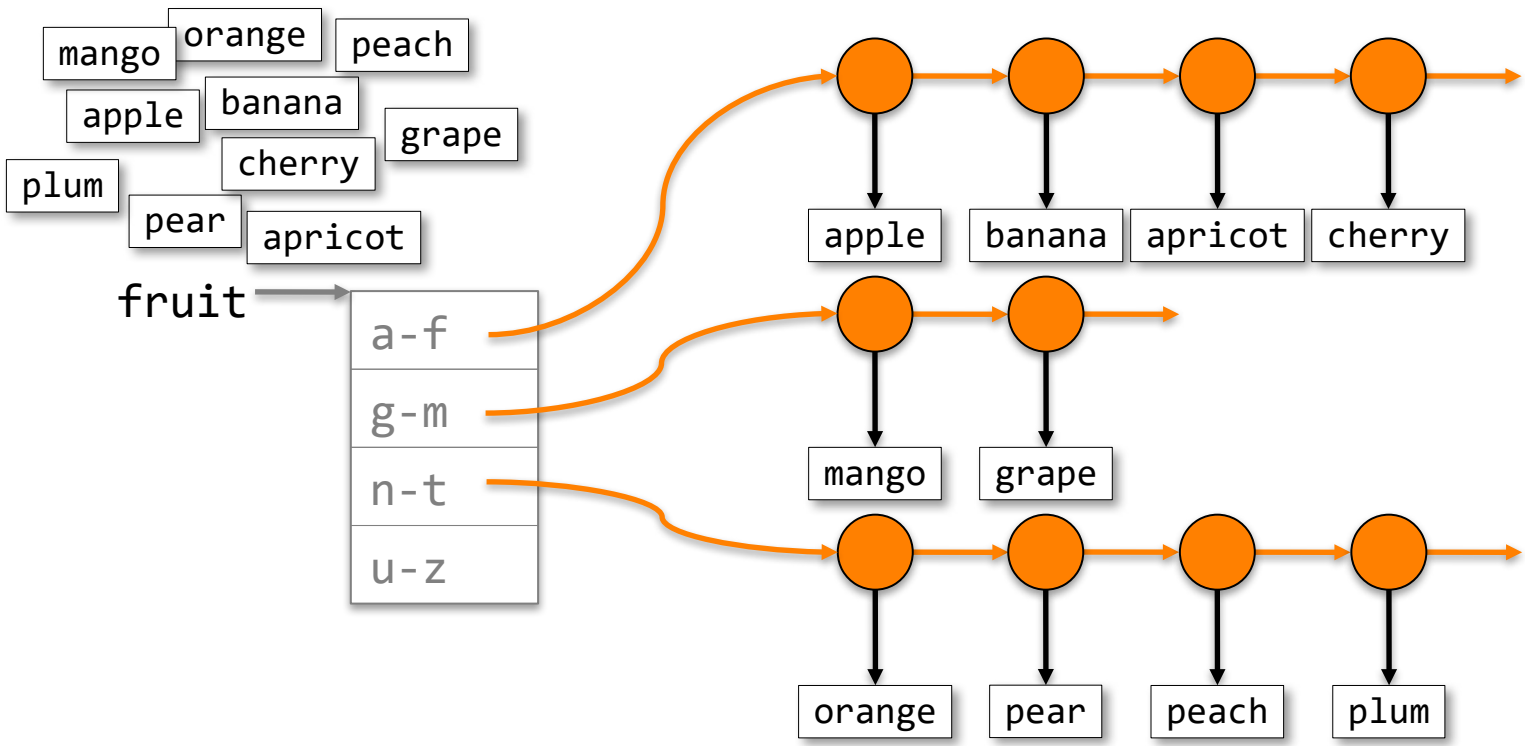
Main challenges are: a) dealing with **hash collisions** and dealing with **load** (how big to make the table).

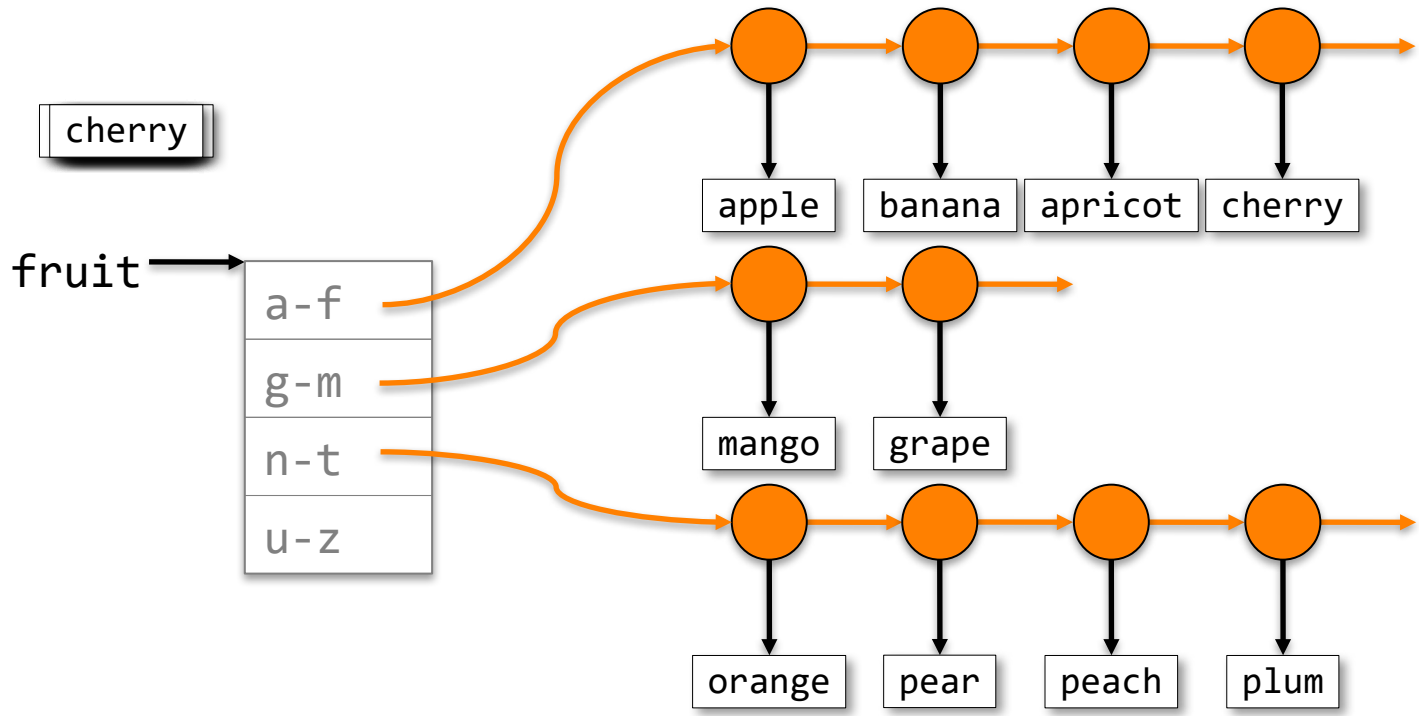
Two broad approaches:

- Separate chaining
 - Hash table entries are lists: (key, value) pairs are in lists.
- Open addressing
 - Hash table entries are (key, value) pairs.
 - Collisions resolved by probing – e.g. find next entry slot

HashSet Implementation of a Set

- Special case of hash table where we only have **key** (it is not associated with any *value*).
- We'll demonstrate **separate chaining** where our lists only needs to store a single item rather than a pair.

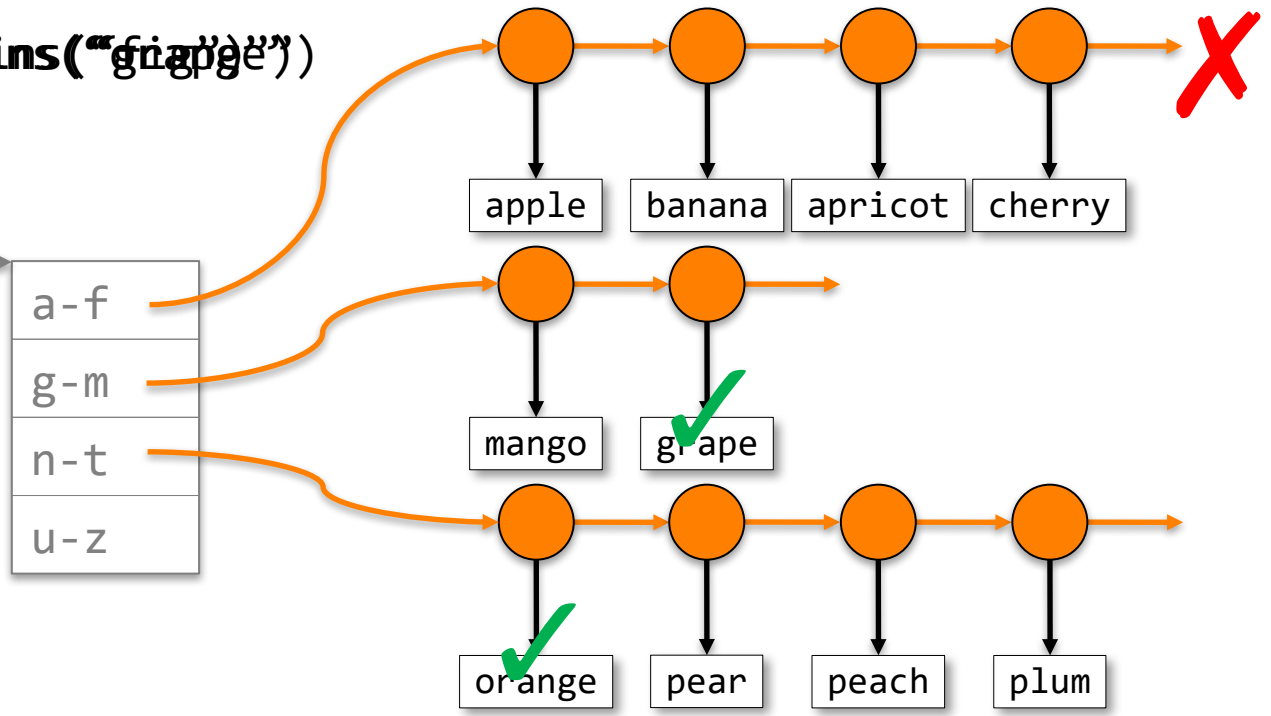




fruit.contains("grape")

grape

fruit



Load Factor

The **load factor** is the ratio of number of elements to the number of “buckets” (size of table).

By resizing (doubling) table capacity when lists grow “too long”, add and contains can run in amortised constant time (assuming a good hash function).

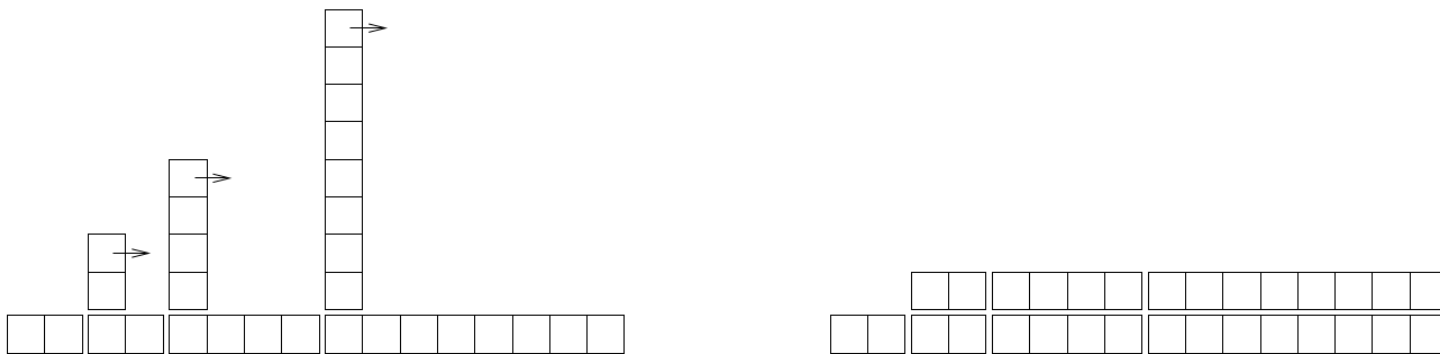


Figure B.1: The cost of a hashtable add.

(Illustration from “Think Python: How to think like a computer scientists” (2nd ed) by Allen B. Downey.)



Complexity

```
boolean add(T value);  
boolean contains(T value);  
int size();  
boolean remove(T value);
```

- add, contains, remove – **Time $O(1)$ amortized, $O(n)$ worst**
 - *good* hash function
 - table resized to keep table *load factor* in a range
- size – **Time $O(1)$**
 - explicitly tracked

Space $O(n)$

A painting of a forest scene with tall, thin trees and a church spire in the distance. The trees have yellow and orange leaves, suggesting autumn. The ground is covered in fallen leaves. In the foreground, there are two figures: one in a dark coat and another in a blue coat. The sky is overcast and grey.

A05 Sets: TreeSet

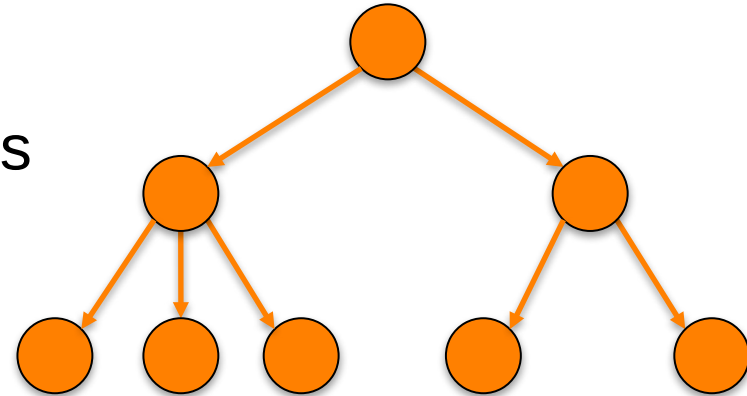
Tree as an ADT
A tree-based Set implementation

Tree as an ADT

The **tree** ADT corresponds to an ordered tree in mathematics.

A tree is defined recursively in terms of nodes:

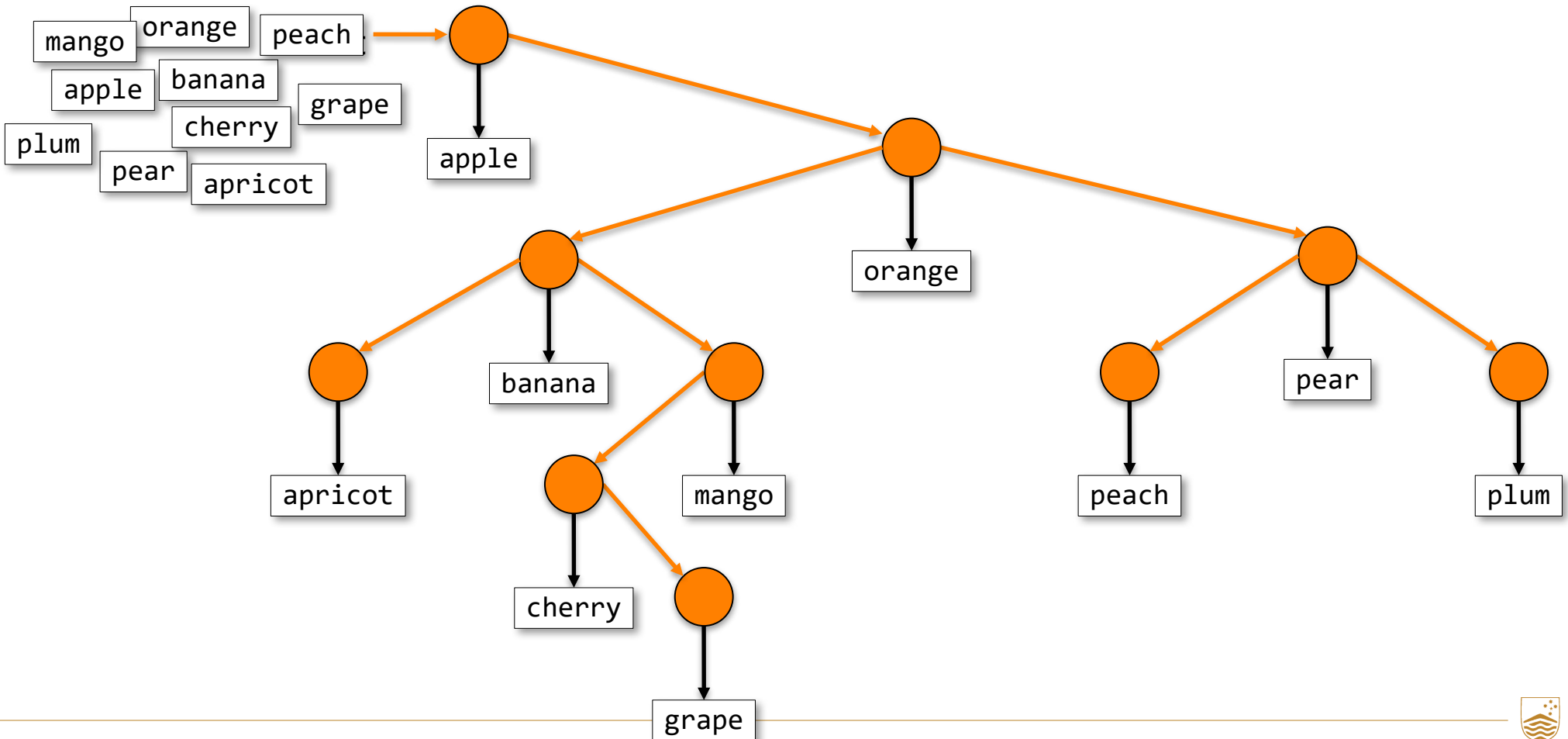
- A tree is a node
- A node contains a value and a list of trees
- No node is duplicated

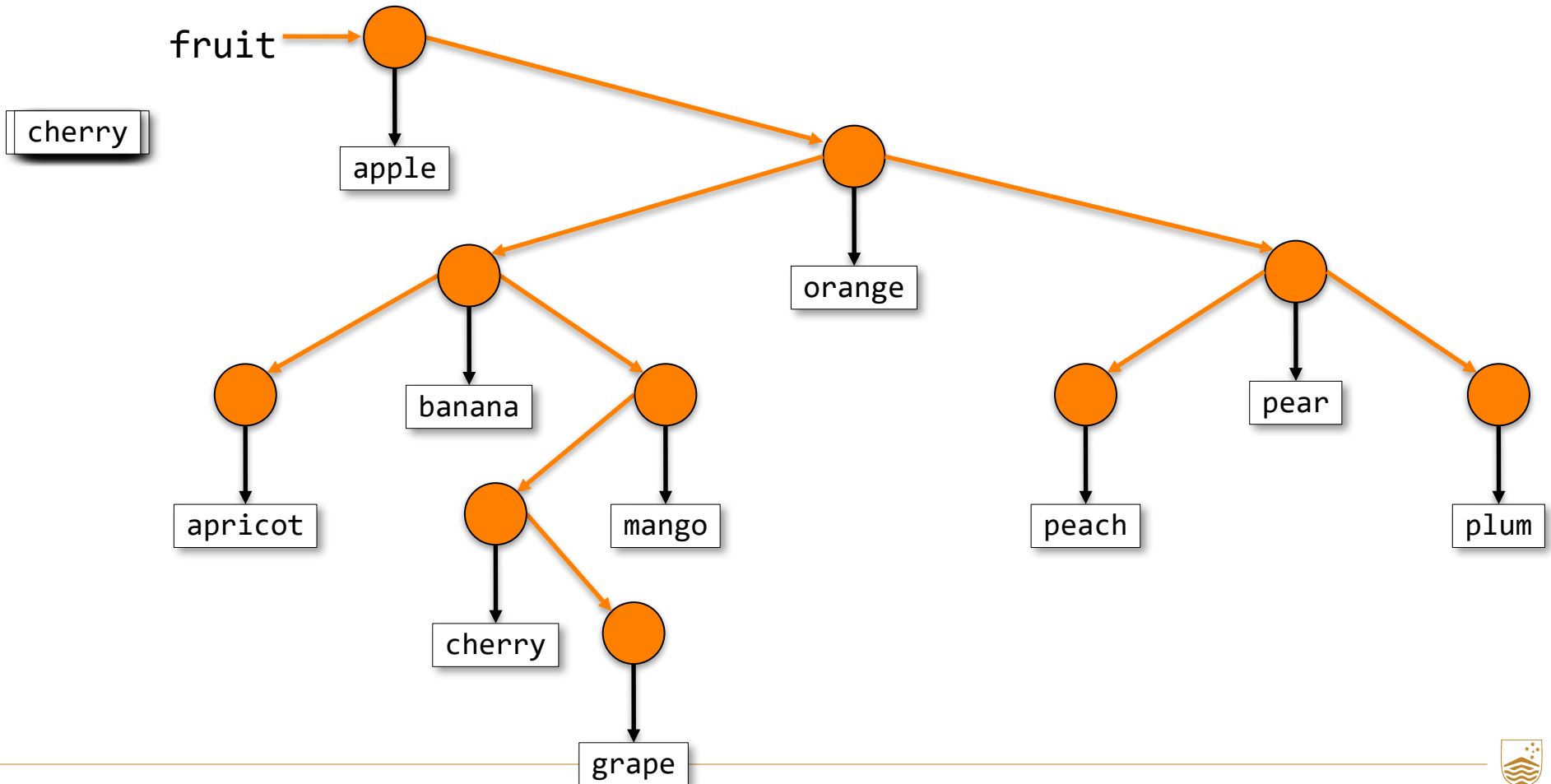


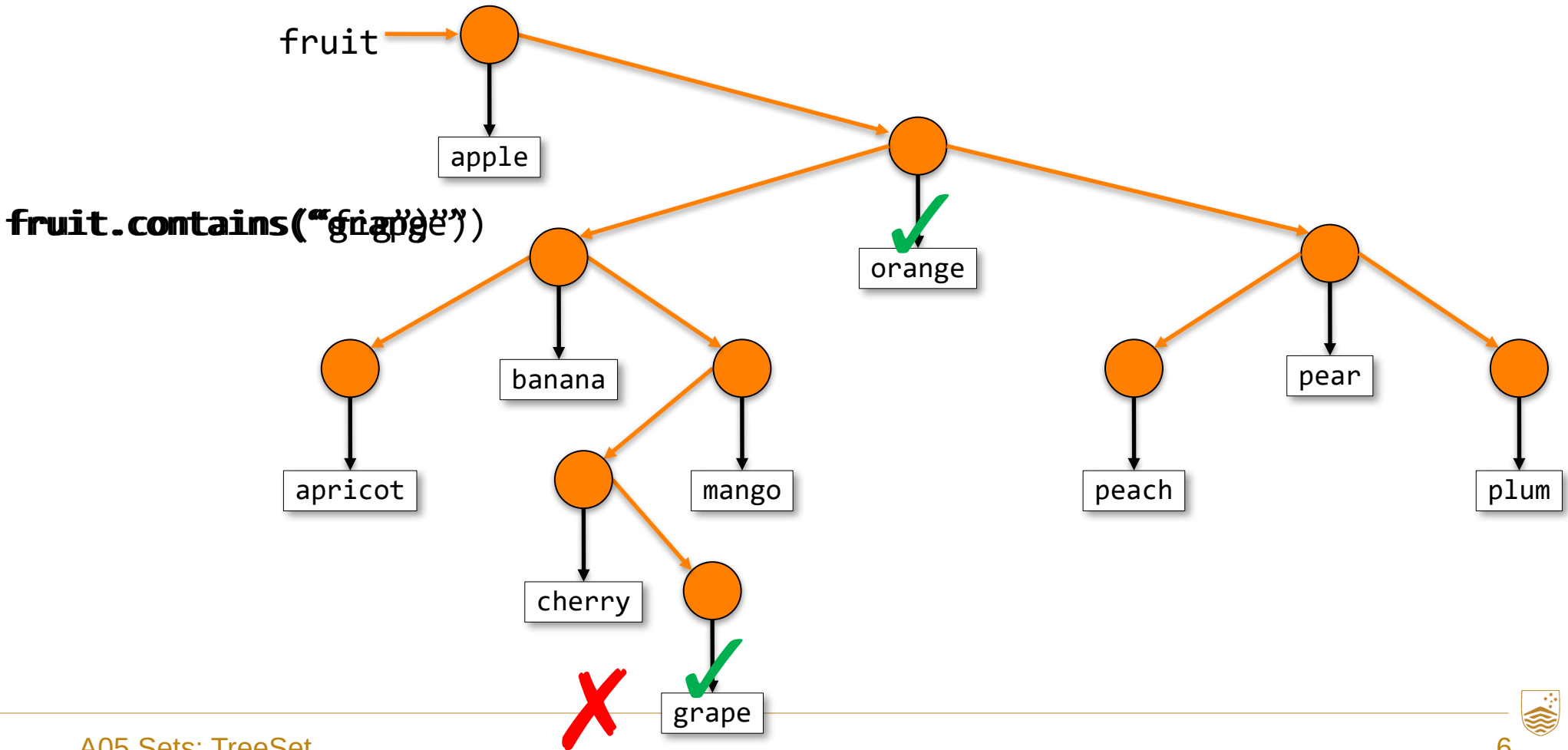
Binary Search Tree to Implement Set

A **binary** search tree is a tree with the following additional properties:

- Each node has at most **two** sub-trees
- Nodes may contain (key, value) pairs, or just keys
- Keys are ordered within the tree:
 - The left sub-tree only contains keys less than the node's key
 - The right sub-tree only contains keys greater than the node's key







Ordering in Java (Recall J14)

Objects of any class that implements the `Comparable` interface can be ordered:

`a.compareTo(b)`

- `< 0` iff a is ordered before b
- `> 0` iff a is ordered after b
- `== 0` if `a.equals(b)` (but also if a and b are not ordered)

Our `Set` interface does not bound our contained type parameter to be `Comparable`, what to do?

- Bound T in the `TreeSet` class declaration:
 - `class TreeSet<T extends Comparable<T>> implements Set<T>`
- Throw runtime exception on use of non-comparable types (the approach in `java.util.TreeSet`).
- Force users to provide `Comparator` (e.g., as lambda expression).

<https://docs.oracle.com/en/java/javase/17/docs/api/java.base/java/lang/Comparable.html>



Complexity

```
boolean add(T value);  
boolean contains(T value);  
int size();  
boolean remove(T value);
```

- add, contains, remove – **Time $O(\log(n))$ amortized, $O(n)$ worst**
 - self-balancing trees (e.g., B-Trees) have $O(\log(n))$ worst case
- size – **Time $O(1)$**
 - explicitly tracked

Space $O(n)$

A painting of a person walking on a path through trees towards a body of water. The scene is rendered in a soft, impressionistic style with visible brushstrokes. The person is in the lower center, walking away from the viewer. The path is sandy and leads through several trees with green and yellow foliage. In the background, there is a large body of water and a distant shoreline with some buildings. The sky is a mix of blue and white, suggesting a bright, slightly overcast day.

A06 Maps: HashMap and TreeMap

Map as an ADT

A Map interface

A hash-table-based Map implementation

A tree-based Map implementation

ADT Recap

First-principles implementation of three Java container types:

- List
 - **ArrayList**, **LinkedList** implementations (A1, A2)
- Set
 - **HashSet**, **TreeSet** implementations (A3, A4, A5)
- Map
 - **HashMap**, **TreeMap** implementations (A6)

Introduced **hash tables**, **trees** (A4, A5)

The Map ADT (also known as Associative Array)

A map consists of **(key, value)** pairs

- Each key may occur only once in the map
- Values are retrieved from the map via the key
- Values may be modified
- Key, value pairs may be removed

Our Map Interface

We will explore maps using a simple interface:

```
public interface Map<K, V> {  
    V put(K key, V value);  
    V get(K key);  
    V remove (K key);  
    int size();  
}
```

```
fruit.get("grape", 3.00)
```

3.00
grape

fruit

