

Path-Sensitive Backward Slicing

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Introduction

- We introduce *path-sensitivity* to *slicing*
 - **Slicing**: Given a *criterion* a control point l and variable x , find **subset of statements** that affects the value of x at l
 - Applications in parallelization, software testing, program comprehension, reverse engineering, etc.
- **Path-sensitivity**: Considers *infeasible* paths for precision
 - Typically considered expensive
- Our approach:
 - *Craig interpolation* to mitigate complexity
 - *Witness paths* to maintain accuracy

(Backward) Program Slicing

```
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⟨1⟩ if (a>=0) c=-a;  
⟨2⟩ if (b>=0) x=y; else c=b;  
⟨3⟩ if (c>0) x++; else x=1; ⟨4⟩
```

Criterion: $\langle\langle 4 \rangle, x\rangle$

We compute a smallest set *INF*:

- 1 Include statements that affect the value of criterion variable
- 2 *Data dependency*: Include statements that affect the value of variables in *INF*
- 3 *Control dependency*: Include the if/while condition in *INF* if body contains statement in *INF*

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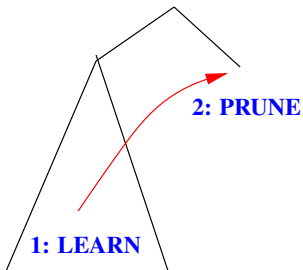
```

Criterion: $\langle\langle 4 \rangle, x\rangle$

- The less statements included the better
- But we included all statements!
- $x=0$ should not be in *INF* as no *feasible* path from $x=0$ to $x++$ s.t. all conditions *satisfied*

Our Solution

- Naive path-sensitive analysis
→ huge search space:
exponential in the size of the program
- To mitigate the problem we employ *learning*



We use information from already traversed (symbolic execution) subtree to prune other subtrees

Symbolic Execution

- We consider program statements as *constraints*
- Analysis by traversal on symbolic execution tree

Example

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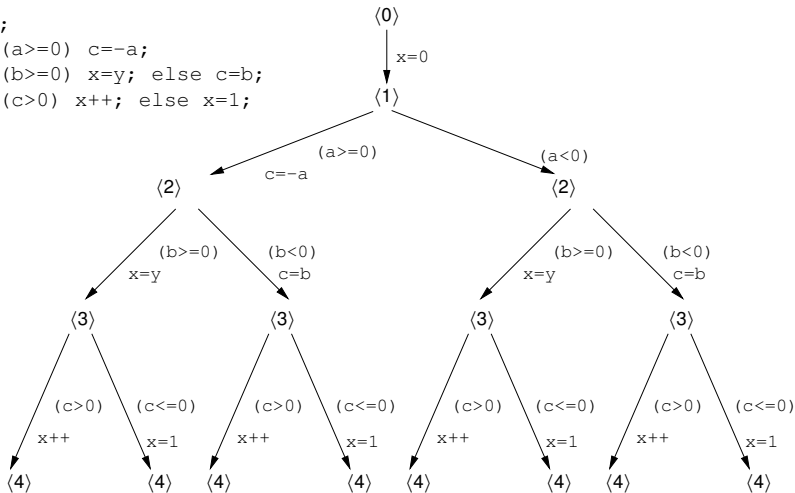
Next: The Tree

Symbolic Execution Tree

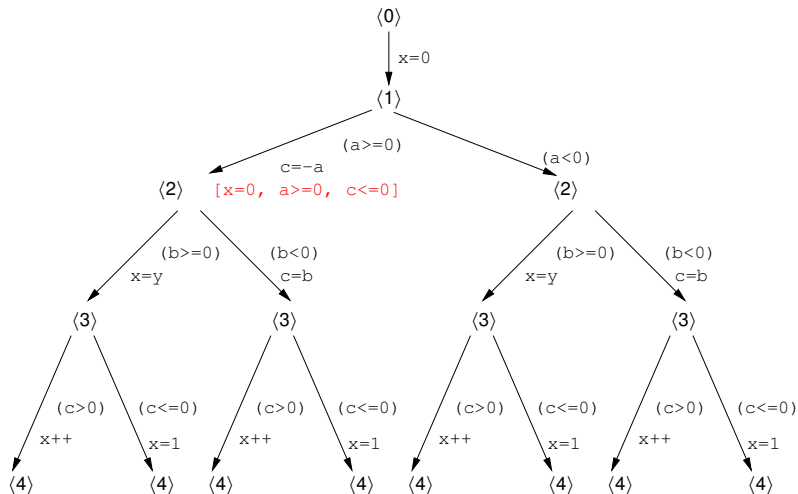
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Symbolic Execution Tree



Symbolic State: constraints on program variables

Interpolation

To reduce search tree size:

Core technology: *Craig Interpolation*

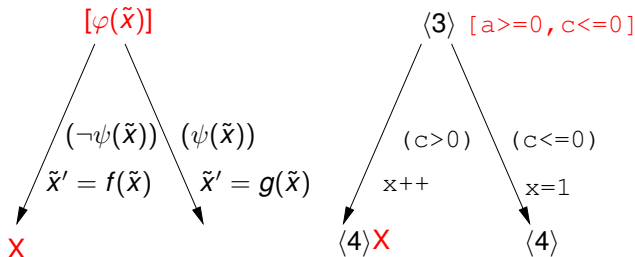
Example:

$$\langle 0 \rangle x = 0; \langle 1 \rangle (a > 0); c = -a; \langle 2 \rangle \\ (b \geq 0); x = y; \langle 3 \rangle (c > 0); x ++; \langle 4 \rangle$$

The last guard is *unsatisfiable* (hence, path is *infeasible*):
remove statements not needed to ensure *unsatisfiability*

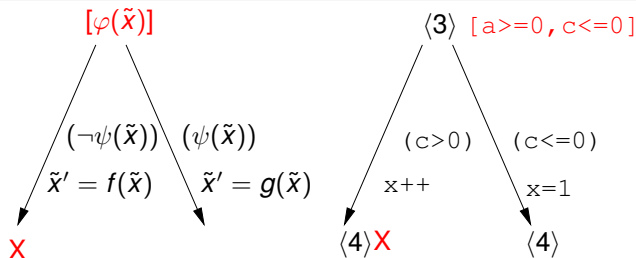
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Logical View



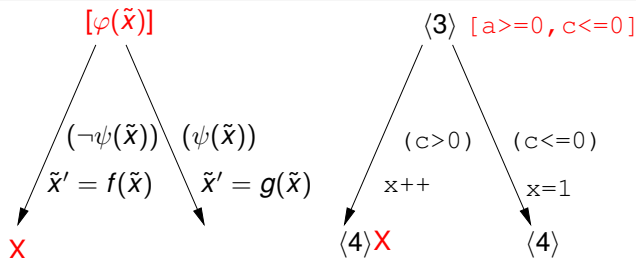
- **Craig interpolant** (e.g., on left path): A condition $\iota(\tilde{x})$ s.t. $\varphi(\tilde{x}) \Rightarrow \iota(\tilde{x})$ and $\iota(\tilde{x}) \Rightarrow (\neg\psi(\tilde{x}) \wedge \tilde{x}' = f(\tilde{x}) \Rightarrow \square)$
- Most general: path's **weakest liberal precondition**:
 Left: $\forall \tilde{x}' : \neg\psi(\tilde{x}) \wedge \tilde{x}' = f(\tilde{x}) \Rightarrow \square$ in example: $c \leq 0$
 Right: $\forall \tilde{x}' : \psi(\tilde{x}) \wedge \tilde{x}' = g(\tilde{x}) \Rightarrow \neg\square$ in example: $\neg\square$
 Conjunction is a **weakest precondition** of **feasible** paths:
 $c \leq 0 \equiv \exists x' : c \leq 0 \wedge x' = 1$
- Its **approximation** efficiently computed by **removal** of $a \geq 0$

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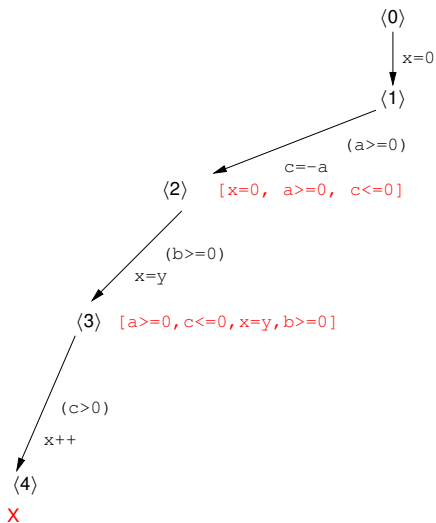
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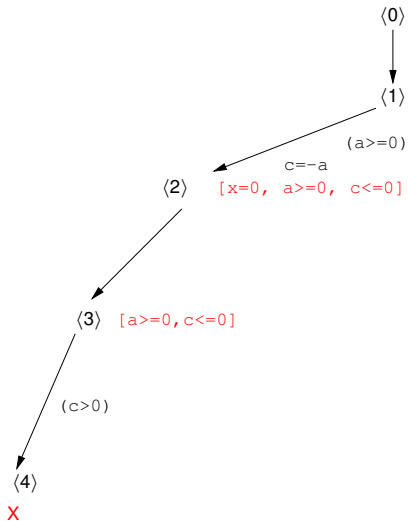


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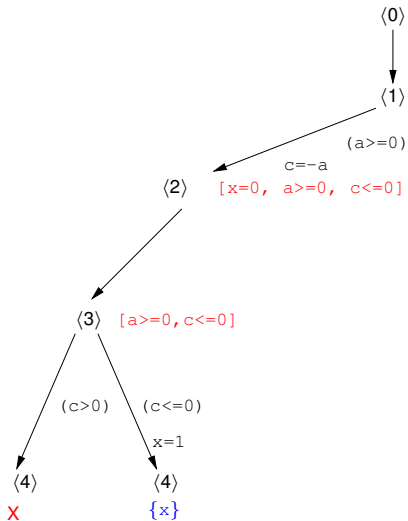
Efficient Path-Sensitive Slicing



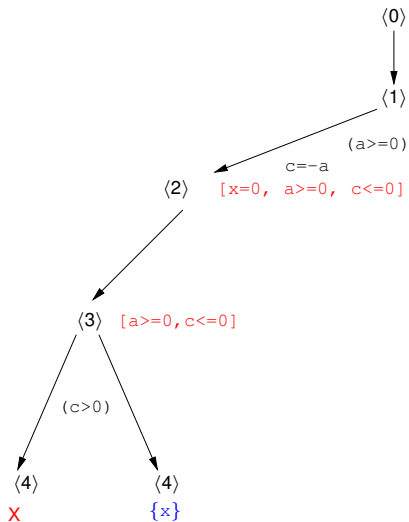
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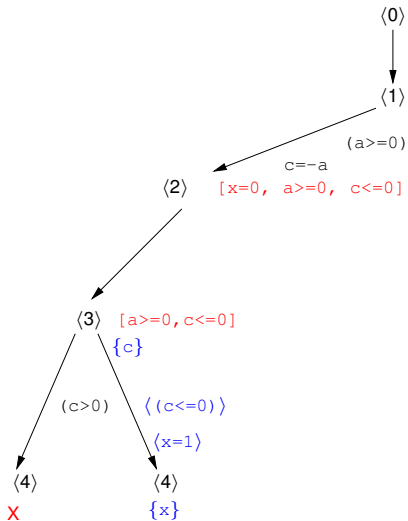
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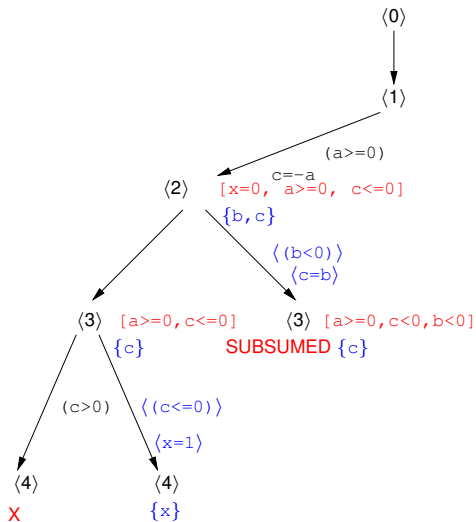
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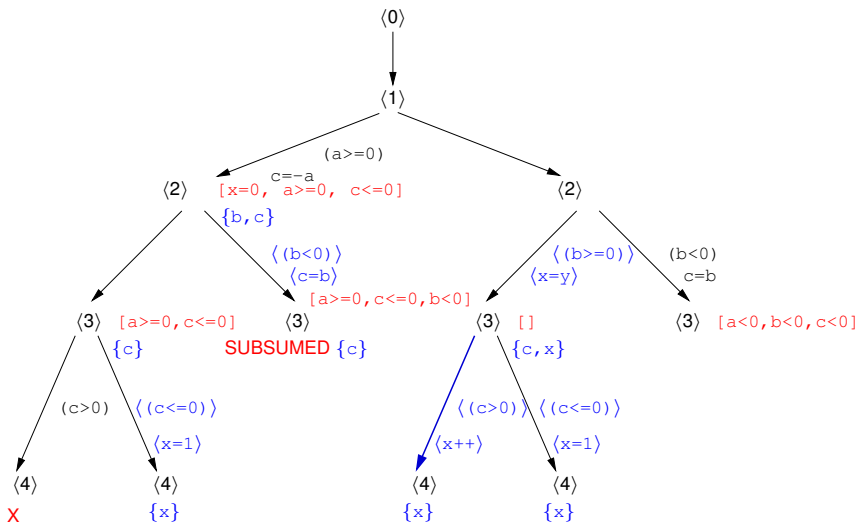
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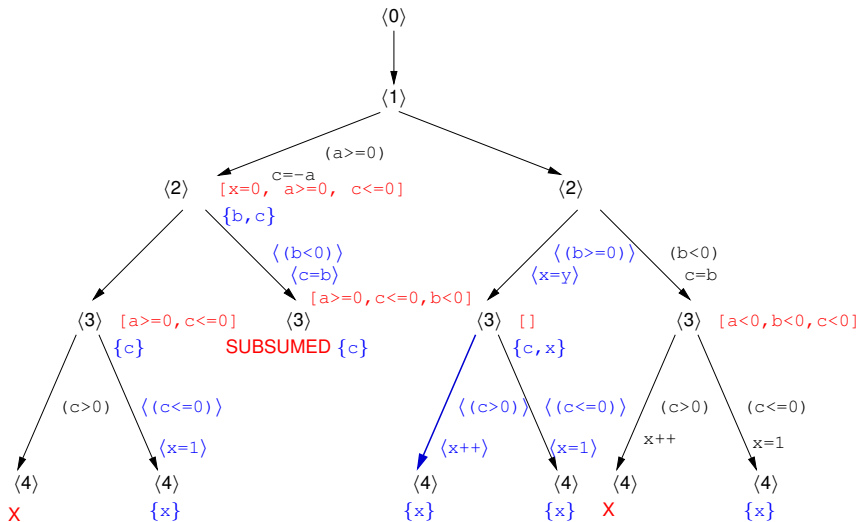
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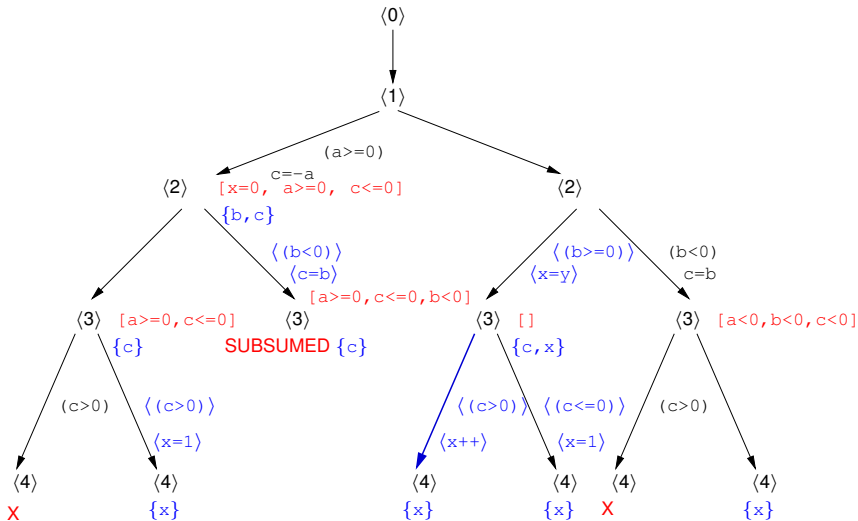
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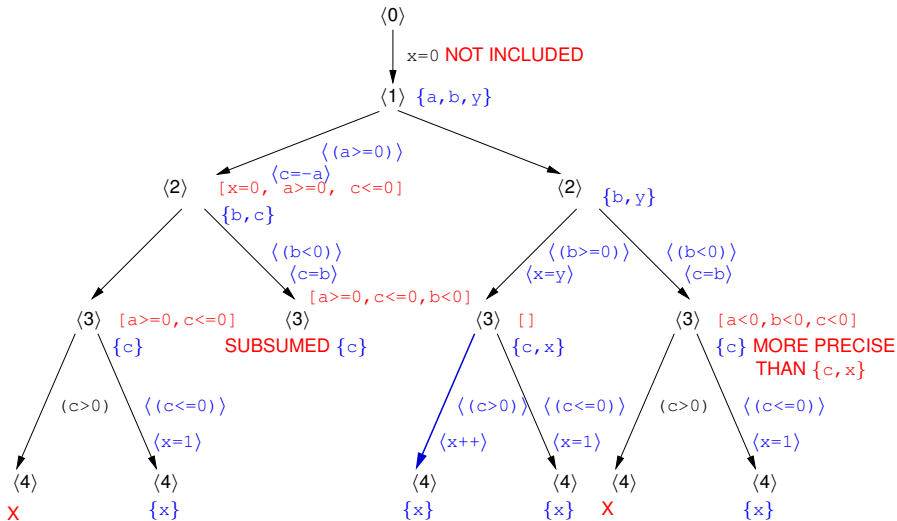
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Loops

- 1 Initial pass to generate loop invariant via *abstract interpretation*.
 - Implementation uses **InterProc**
 - Example (next) uses removal of constraints of updated variables
- 2 The loop invariant used in symbolic execution
- 3 Fixpoint computation to generate backward dependencies

Loops

```

<0> i=10;
<1> while (i>0) {
<2>     if (x>y) x--;
        i--;
    } <3>

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<0> [y=3]

↓ i=10

<1> [y=3, i=10]

Criterion:

<<3>,x>

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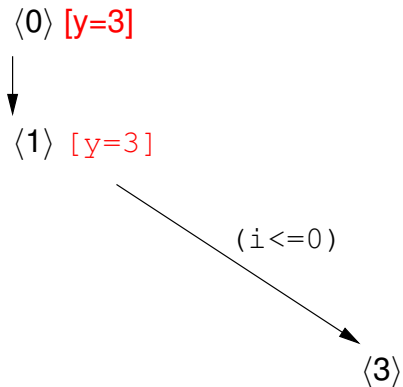
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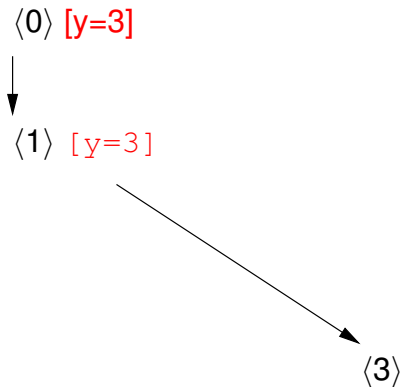
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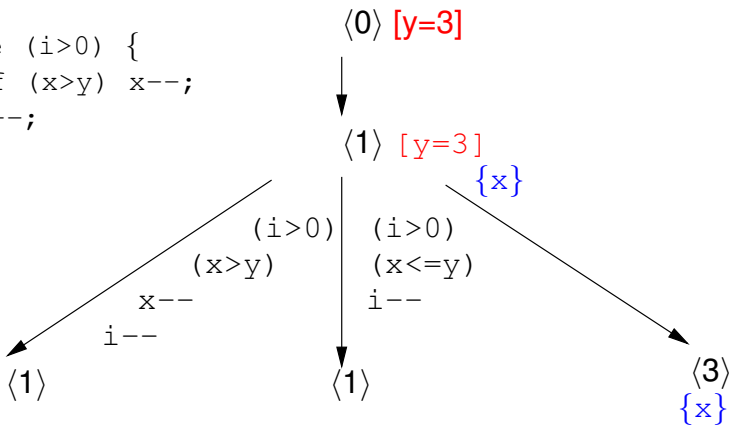
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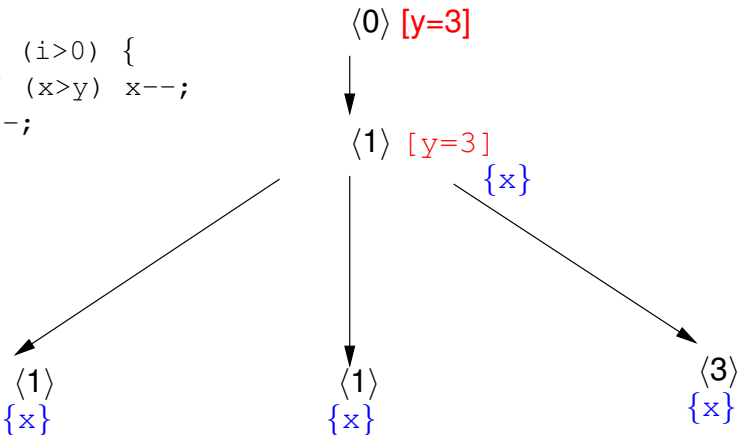


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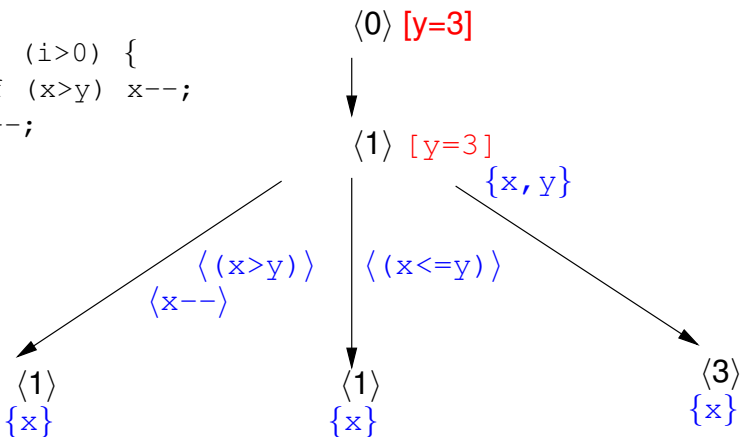
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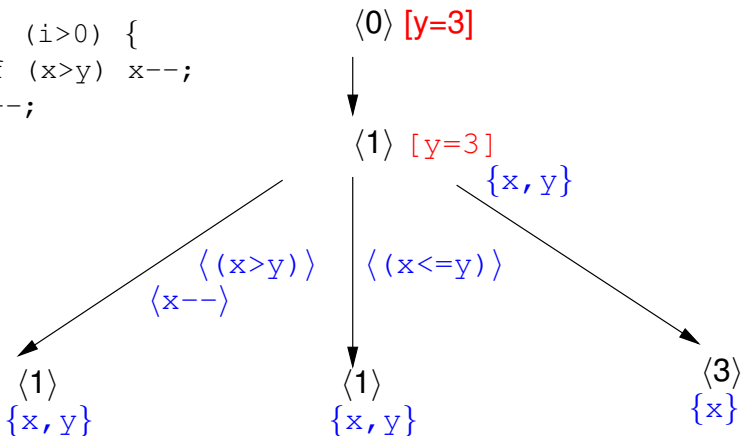
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Experimental Results

Program	LOC	Path-Insens		Path-Sens		
		Size Red	Time	Reuse		No Reuse
				Size Red	Time	Time
mpeg	5K	4%	21s	8%	628s	∞ ¹
diskperf	6K	32%	2s	57%	94s	∞
floppy	8K	36%	9s	47%	263s	∞
cdaudio	9K	23%	10s	52%	301s	∞
serial	12K	39%	16s	50%	395s	∞
fcron.2.9.5	12K	42%	32s	61%	832s	∞
Mean		23%	15s	38%	418s	—

Results on Intel 3.2GHz 2Gb. ¹ timeout after 2 hours or 2.5 Gb of memory consumption

References

- J. Jaffar, V. Murali, J. A. Navas, and A. E. Santosa. *Path-Sensitive Backward Slicing*. SAS '12
- J. Jaffar, V. Murali, J. A. Navas, and A. E. Santosa. *TRACER: A Symbolic Execution Tool for Verification*. CAV '12.
- J. Jaffar, J. A. Navas, and A. E. Santosa. *Unbounded Symbolic Execution for Program Verification*. Proc. RV '11.
- J. Jaffar, A. E. Santosa, and R. Voicu. *An Interpolation Method for CLP Traversal*. In 15th CP, Springer, 2009
- J. Jaffar, A. E. Santosa, and R. Voicu. *Efficient Memoization for Dynamic Programming with Ad-Hoc Constraints*. In 23rd AAI, pages 297–303, AAAI Press, 2008

Related Work

- M. Weiser. *Program Slicing*. In ICSE '81
- Canfora, Cimitile, de Lucia. *Conditioned Program Slicing*. Information and Software Technology 40, 1998
- Danicic, Fox, Harman. *Consit: A Conditioned Program Slicer*. ICSM '00
- Daoudi et al. *Consus: A Scalable Approach to Conditioned Slicing*. Working Conference on Reverse Engineering, 2002.
Scalable = infeasible path detection. This we already do
- Snelting. *Combining Slicing and Constraint Solving for Validation of Measurement Software*. SAS '96.
Less precise: Conservatively includes a statement if there's at least one feasible path to the criterion point

Conclusion

- Mitigating state-space blowup using interpolation
- Application in path-sensitive slicing
- Applicable to other analyses
 - Program verification
 - Worst-case execution time (WCET)
 - Memory usage bound
 - Automatic symmetry reduction
 - Dynamic programming
 - Automatic parallelization (?)

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