Type Inference for the Spine View of Data

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The Question

Can you add the spine view of data to a Hindley-Milner System without sacrificing type inference?

The Answer

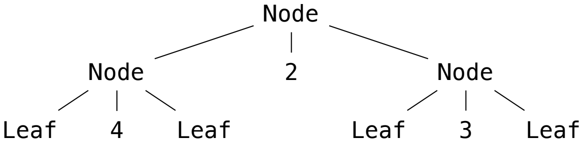
Yes!

The Context

- It has been shown already that extensions of Hindley-Milner which include type annotations can compute types for the spine view[1] of data
  - bondi (The Pattern Calculus) [2]
  - Scrap Your Boilerplate (GHC) [4]
  - DGEN[5]

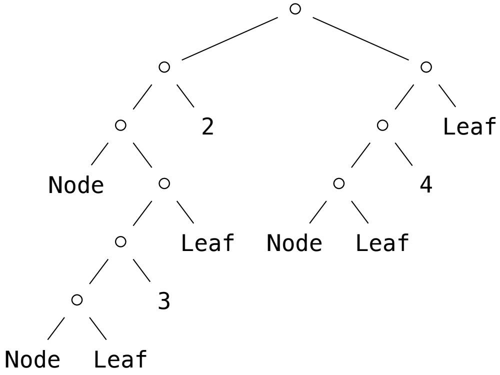
The Usual View of Data

```
data Tree a = Node (Tree a) a (Tree a) | Leaf
tree1 = (Node (Node Leaf 4 Leaf) 2 (Node Leaf 4 Leaf))
```



The Spine View of Data

```
data Tree a = Node (Tree a) a (Tree a) | Leaf
tree1 = (Node (Node Leaf 4 Leaf) 2 (Node Leaf 4 Leaf))
```



Example Term

Apply a function to every sub-tree of a tree a.k.a *Apply a function to every sub-term of a term*

bondi

```
let rec (apply2all : (all a. (a -> a)) -> b -> b) f  z = (
  | Ref x as y -> f y
  | x y -> f ((apply2all f x) (apply2all f y))
  | x -> f x)
z
;;
```

Example Term

Apply a function to every sub-tree of a tree a.k.a *Apply a function to every sub-term of a term*

Scrap Your Boilerplate

```
everywhere :: (forall a. Data a => a -> a)
           -> (forall a. Data a => a -> a)
everywhere f = f . gmapT (everywhere f)
```

Example Term

Apply a function to every sub-tree of a tree a.k.a *Apply a function to every sub-term of a term*

dgen

```
def apply_to_all(f,g) :: (forall a . (a) -> a, b) -> b =
    case [g] of
      { [c(a)] -> f(@apply_to_all(f,c)(apply_to_all(f,a)))
      ; [o]    -> f(o)
      } otherwise -> error "partial definition error in
          apply_to_all"
```

A Single Language Addition Supports the Spine View

```
ispair e bind (x,y) in f else g
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A Single Language Addition Supports the Spine View

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```

Can we devise an algorithm (in the style of Hindley-Milner) which can correctly elaborate the types for any program in a language with no type annotations on terms and which *includes the `ispair` term*?

The Key Contribution

A local implementation of existential types in the type inference algorithm (which requires only constants in the unification algorithm) is enough to support `ispair` in the Hindley-Milner type inference algorithm.

A Hindley-Milner System - FCP

Type Language

$$
\begin{array}{rcll}
\textbf{type schemes } \sigma & ::= & \forall \alpha . \sigma & \text{(quantified type)} \\
 & | & \tau & \text{(monotype)} \\
\textbf{monotypes } \tau, \rho & ::= & \alpha & \text{(type variables)} \\
 & | & T\, \tau_1 \ldots \tau_n & \text{(constructed types)} \\
 & | & \tau \to \rho & \text{(function types)}
\end{array}
$$

A Hindley-Milner System - FCP

Term Language

$$
\begin{aligned}
\textbf{expressions}\ e, f, g ::= &\ v \\
\mid &\ e\ f \\
\mid &\ \texttt{letrec}\ x = e\ \texttt{in}\ f \\
\mid &\ \texttt{case}\ e\ \texttt{of}\ (K(x_1, \ldots, x_n)) \rightarrow f
\end{aligned}
$$

$$
\begin{aligned}
\textbf{values}\ v ::= &\ \lambda x.e \\
\mid &\ K(v_1, \ldots, v_n) \\
\mid &\ s
\end{aligned}
$$

$$
\begin{aligned}
\textbf{semi-values}\ s ::= &\ x \\
\mid &\ s\ v
\end{aligned}
$$

A Hindley-Milner System - FCP_ς

Term Language

$$
\begin{aligned}
\textbf{expressions}\ e, f, g ::=&\ v \\
|&\ \ e\ f \\
|&\ \ \texttt{letrec}\ x = e\ \texttt{in}\ f \\
|&\ \ \texttt{case}\ e\ \texttt{of}\ (K(x_1, \ldots, x_n)) \rightarrow f \\
|&\ \ \texttt{ispair}\ e\ \texttt{bind}\ (x, y)\ \texttt{in}\ f\ \texttt{else}\ g \\
\textbf{values}\ v ::=&\ \lambda x.e \\
|&\ \ K(v_1, \ldots, v_n) \\
|&\ \ s \\
|&\ \ \pi \\
\textbf{semi-values}\ s ::=&\ x \\
|&\ \ s\ v
\end{aligned}
$$

Type System

$$\frac{A \vdash e\colon \tau_e \qquad A_{x,y}, x\colon \alpha \to \tau_e, y\colon \alpha \vdash f\colon \tau \qquad A \vdash g\colon \tau \qquad \alpha \notin TV(A, \tau, \tau_e)}{A \vdash (\texttt{ispair}\ e\ \texttt{bind}\ (x, y)\ \texttt{in}\ f\ \texttt{else}\ g)\colon \tau}$$

Inference Rule

$$\frac{\begin{array}{c} TA \vdash c\colon \tau_c \ \text{mod}\ V \\ T'\,T(A, x\colon \alpha \to \tau_c, y\colon \alpha) \vdash t\colon \tau_t \ \text{mod}\ (V \cup \{\alpha\}) \\ T''\,T'\,TA \vdash e\colon \tau_e \ \text{mod}\ V \qquad \tau_t \overset{U}{\sim} \tau_e \ \text{mod}\ V \\ \alpha \text{ new} \qquad \alpha \notin TV(UTA, U\tau_t) \end{array}}{UT''\,T'\,TA \vdash \texttt{ispair}\ c\ \texttt{bind}\ (x, y)\ \texttt{in}\ t\ \texttt{else}\ e\colon U\tau_e \ \text{mod}\ V}$$

Evidence that these rules are correct

- Type system is sound and complete w.r.t the semantics
- Type inference algorithm is implemented and heavily exercised in DGEN
- Proof of correctness of inference algorithm is pending ...

The Consequences

- Any functional language with programmer defined data-types (Haskell, ML, F#, etc.) can support the spine view of data without requiring any type annotations on terms
- FCP [3] has all the required machinery in support of *first-class polymorphism*. Thus the spine view can be a small addition to this already small extension of Hindley-Milner.

Where to now?

- A proof of the type inference algorithm
- DGEN

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