A Relational Approach to Context-Free Language Reachability

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CFL-R

The CFL-R Context

- Computational engine
- Extends context-free language recognition for graphs [Yannakakis, 1990]
  - Find paths in an edge-labeled graph
  - Spell words in the input language
- Wide range of applications for CFL-R:
  - Program analysis [Reps, 1998]
  - Logic programming [Yannakakis, 1990]
  - Formal verification [Dolev et al., 1982]

Example - Points-To Analysis

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- Necessary for many optimisations and other analyses
- Which variables point-to which heap-locations at runtime

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x = malloc(Foo)
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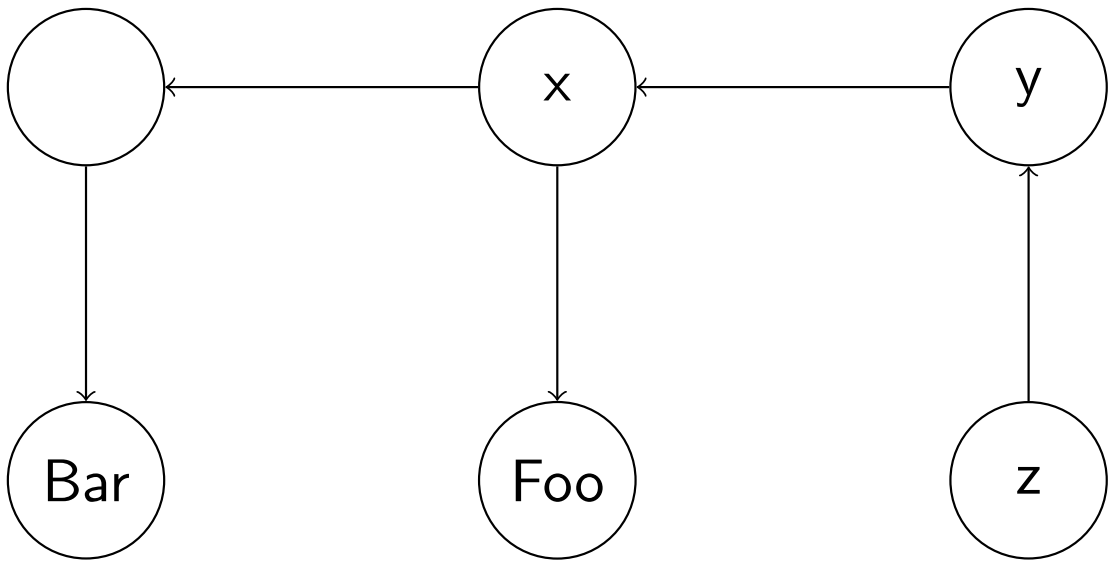

Example - Points-To Analysis

- Transitive closure/Reachability
- Imprecise: over-approximates the correct answer
- Program semantics can be used to improve accuracy

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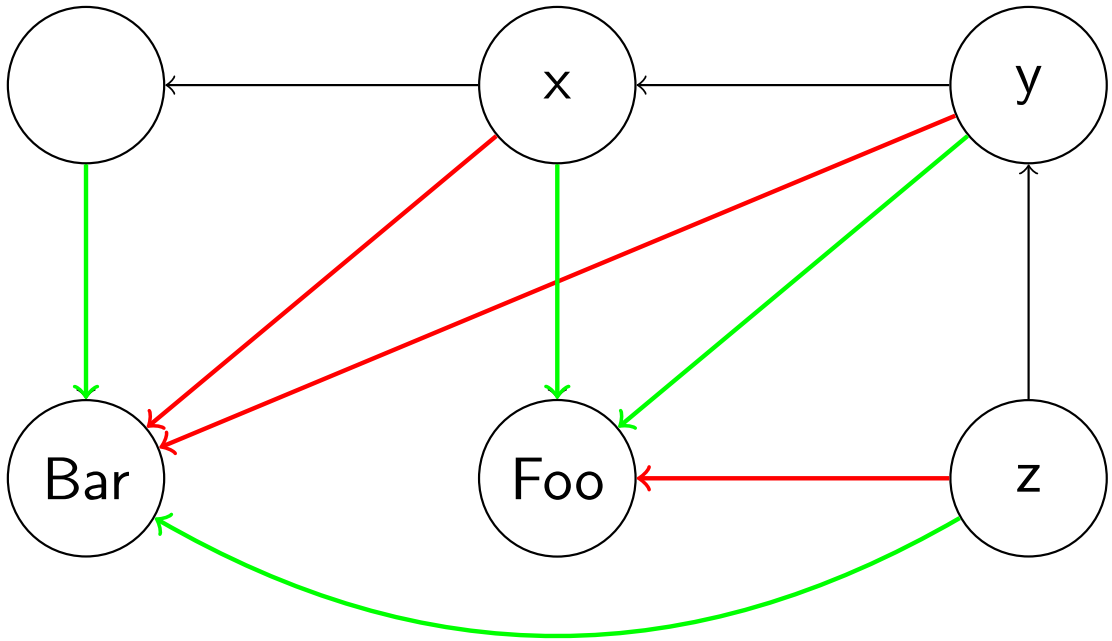

Example - Points-To Analysis

- Label edges
- Paths spell a word
- Words in the language

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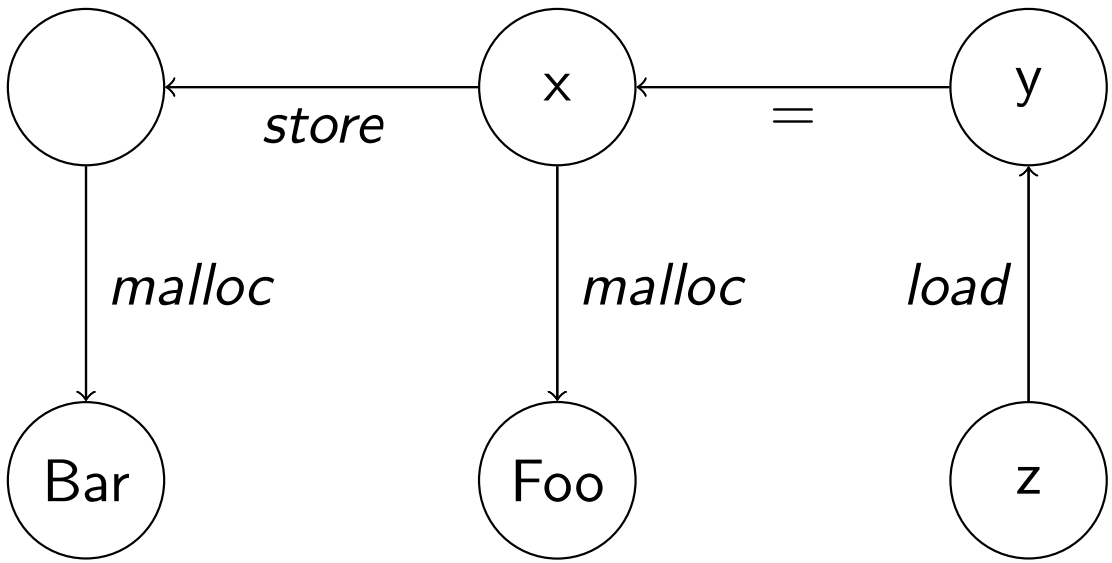

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- Words in the language

- $PT \rightarrow malloc \mid = PT \mid \approx PT$
- $alias \rightarrow PT\ \overline{PT}$
- $\approx \rightarrow load\ alias\ store$

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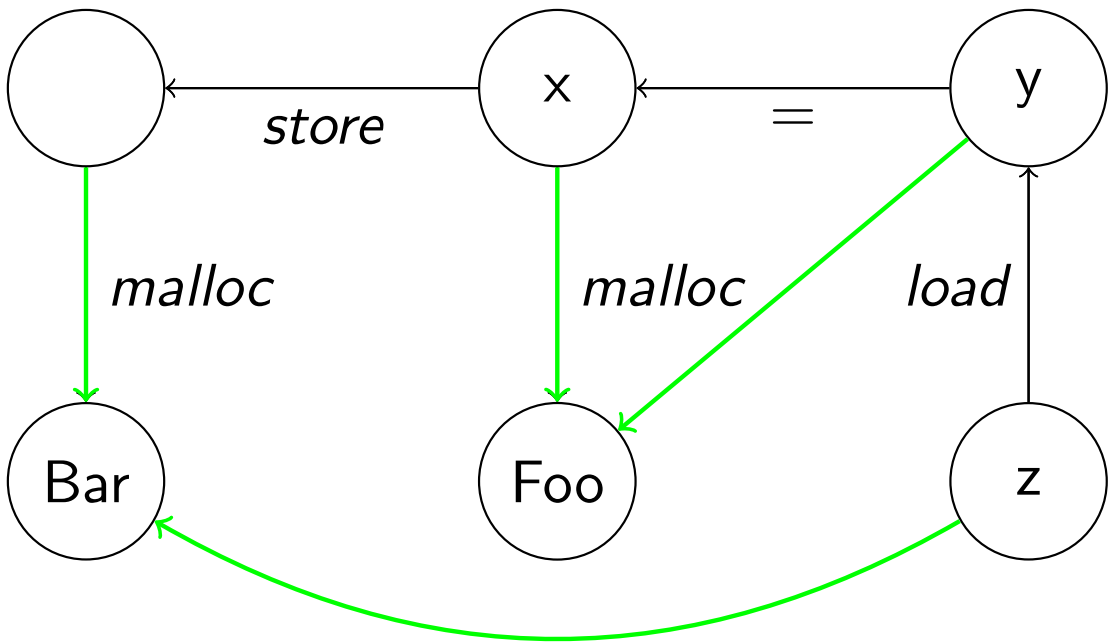

State-of-the-Art Algorithms

- Melski and Reps [Melski and Reps, 2000]
  - Worst case runtime complexity: $\mathcal{O}(n^3)$, $(n = |V|)$
  - Dynamic programming with path-summarisation

- Chaudhuri [Chaudhuri, 2008]
  - Worst case runtime complexity: $\mathcal{O}(\frac{n^3}{\log n})$
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  - Logarithmic speedup only
  - Significant memory cost, requires $\Theta(n^2)$ storage

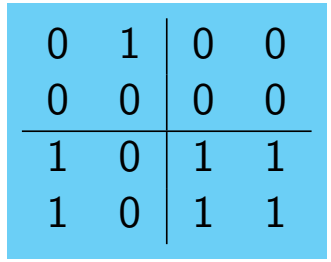
A Relational Approach

Semi-naïve Evaluation

- Adapted from the datalog context
- Use recently discovered (Δ) paths to compute new paths
  - $A \to BC$
  - New Δ_A relations come from at least one Δ_B, Δ_C
- Order the choices of Δ to avoid eager computation
  - $A \to BC$, choose Δ_B, Δ_C before Δ_A
- Conclude when no new paths are discovered
- Depends on **relational operations**
  - union, intersection, difference, identity
  - composition (join)

Relations - Quadtrees

$$\begin{matrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{matrix}$$



- Input edges encoded in an adjacency matrix
- Quadtrees to represent adjacency matrices
- Sparse representation, performs well for sparse matrices
- Matrix multiplication can be used to join paths, $\mathcal{O}(m^2 n \log n)$

Relations - Quadtrees

0	1	0	0
0	0	0	0
1	0	1	1
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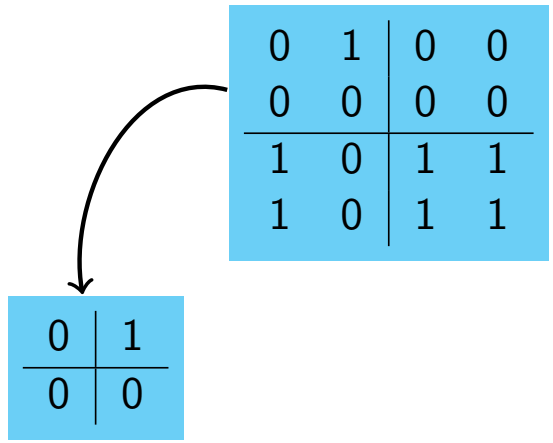

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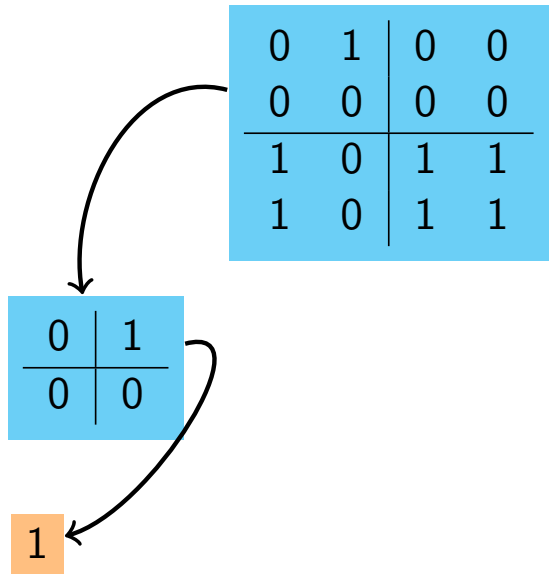

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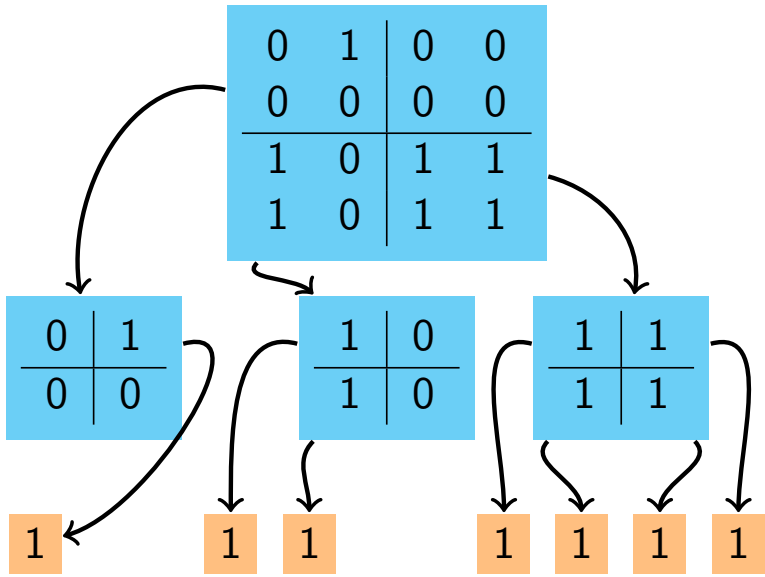

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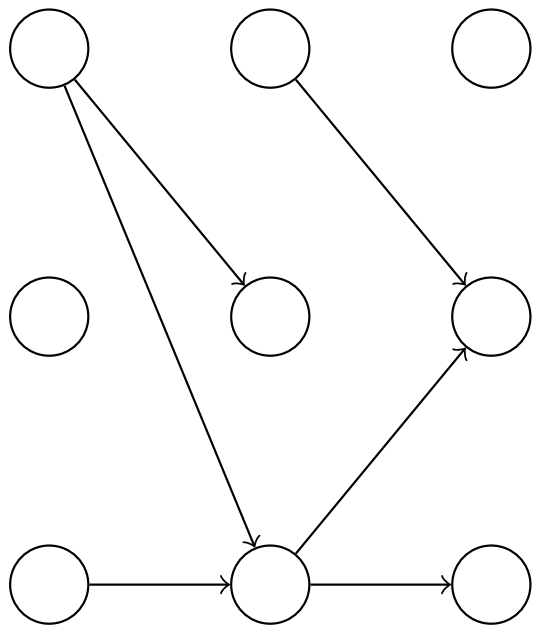

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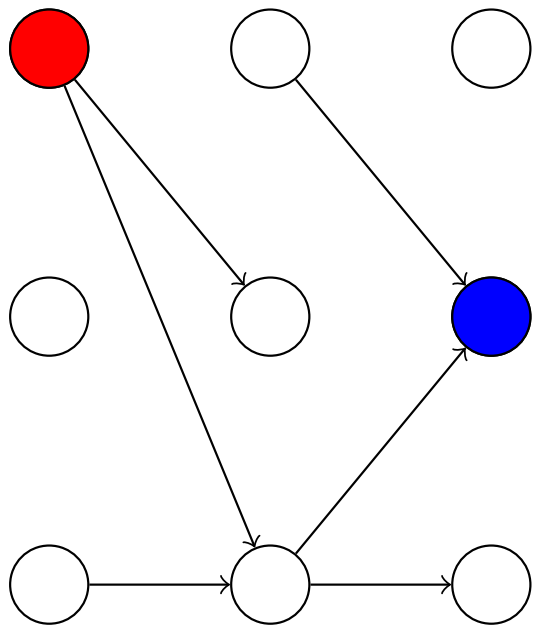

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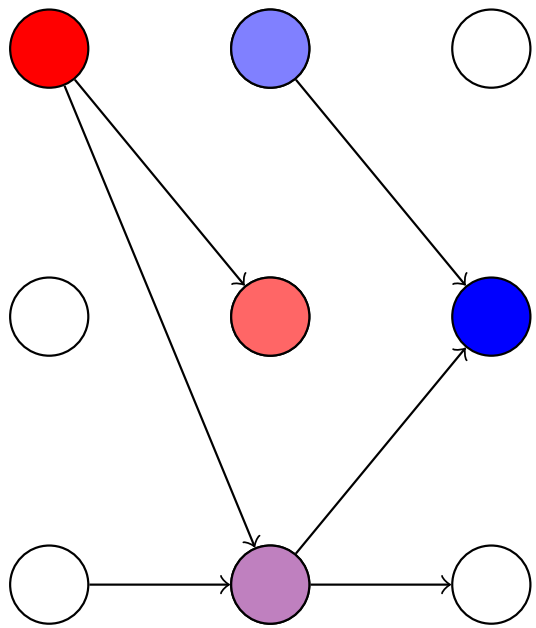

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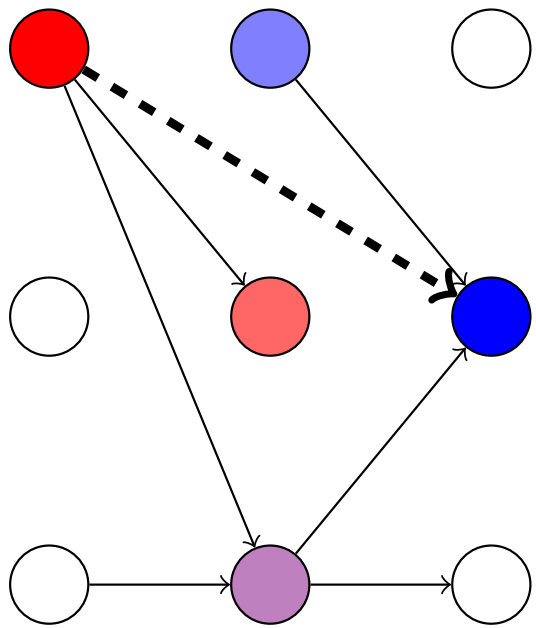

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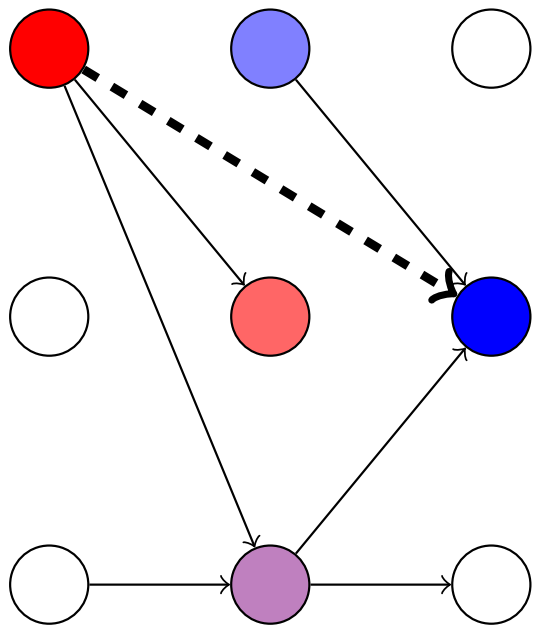

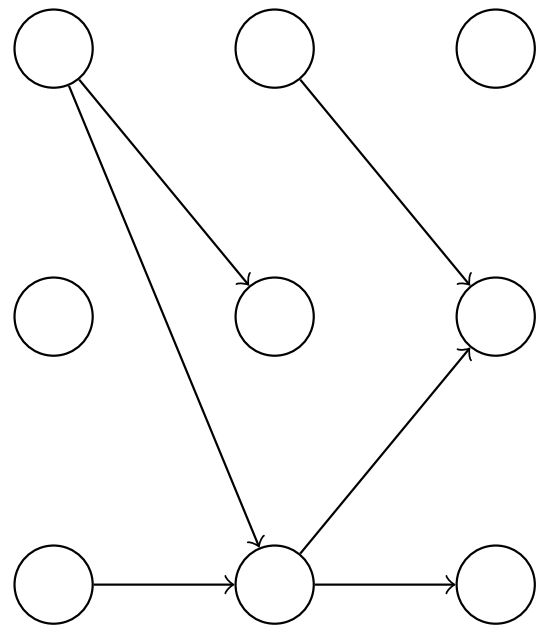

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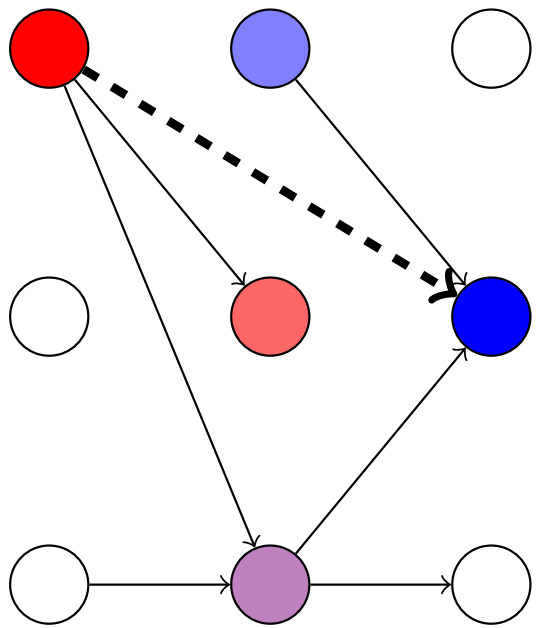

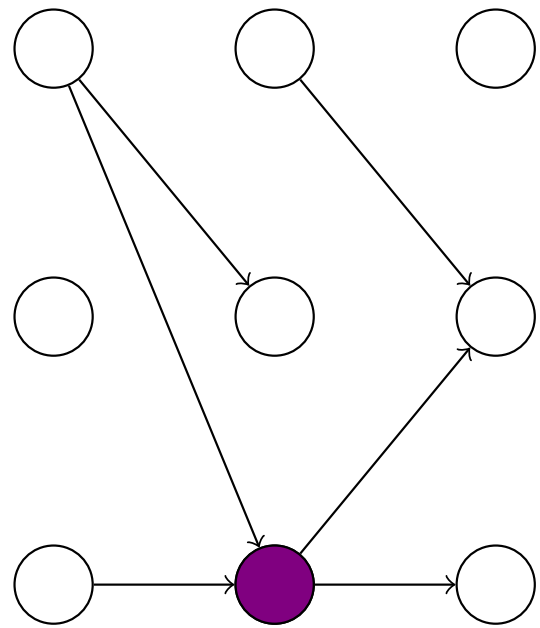

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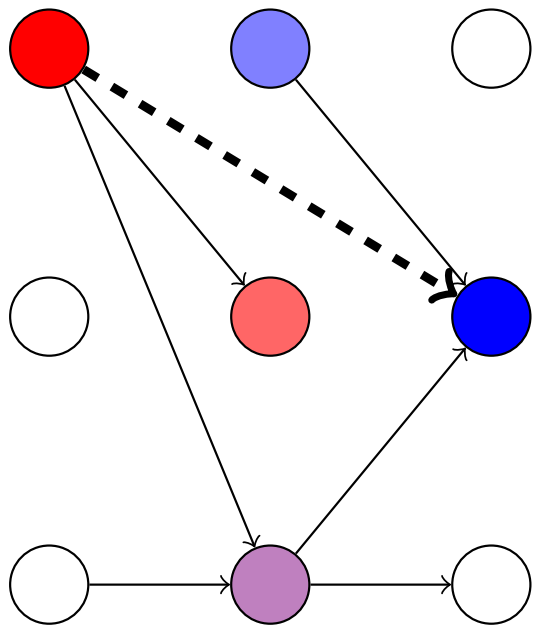

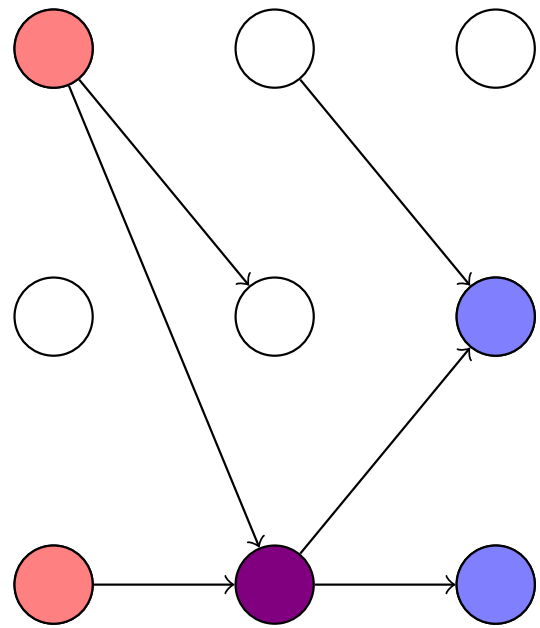

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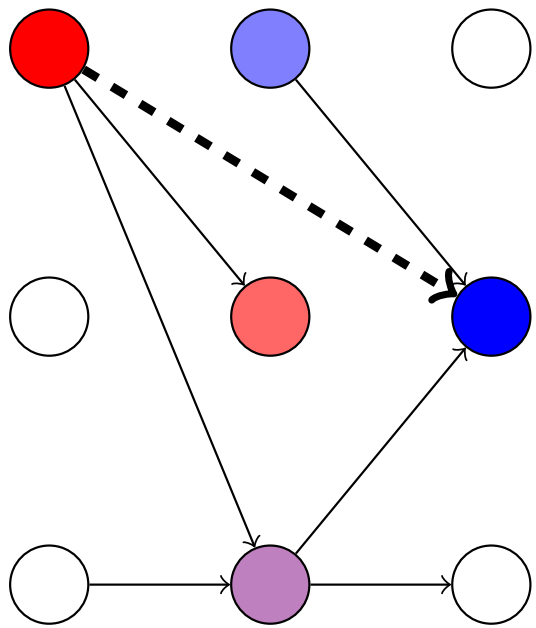

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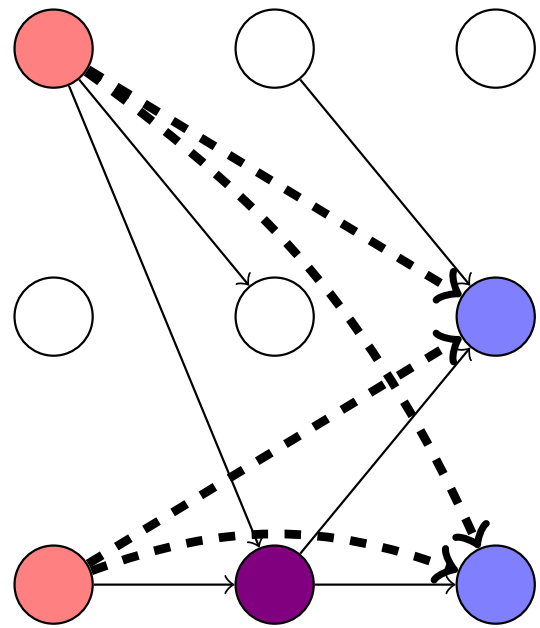
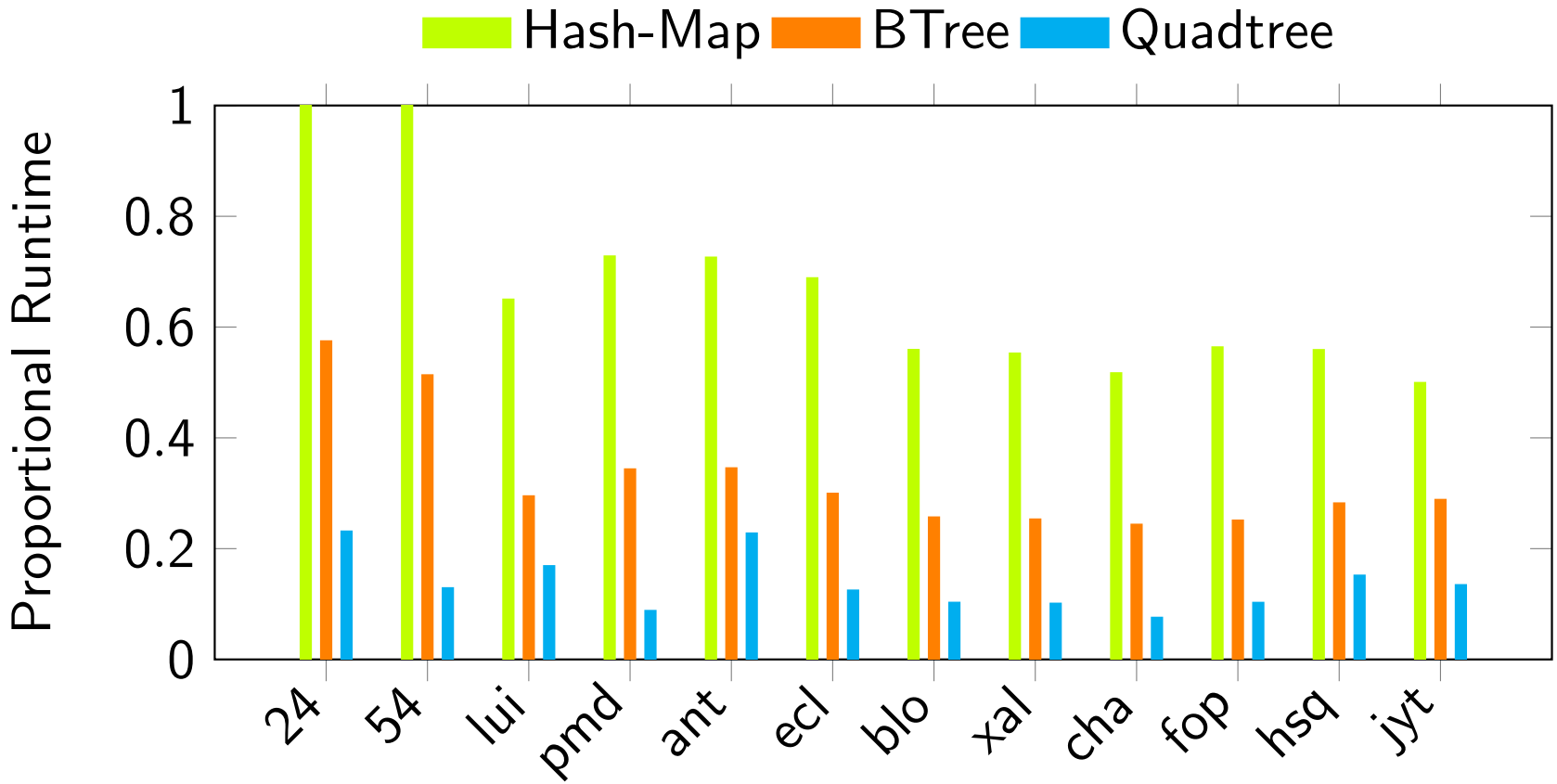


- $\mathcal{O}(m^2 + n)$

Experimental Results

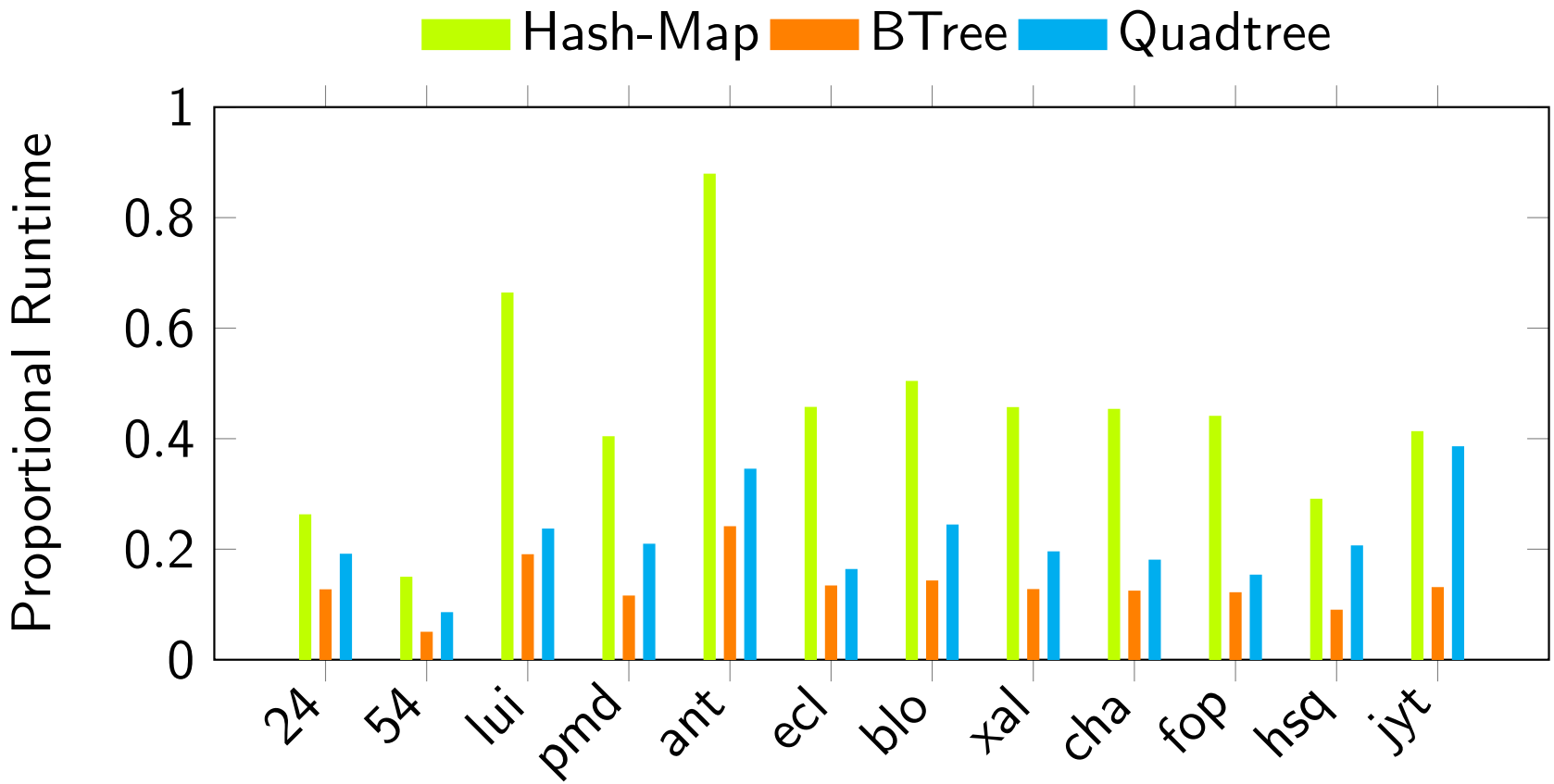
Speedup vs WL

- Significant improvements, especially for larger benchmarks
- Quadtree universally superior



Speedup vs WL

- Improved memory, memory/time tradeoff
- B-Tree implementation is superior



Summary

- CFL-R: Useful computational framework
  - Points-to, security, model checking
- Current algorithms are ineffective
  - Inefficient/redundant computations, $\mathcal{O}(n^3)$
- Relational approach
  - Semi-naïve: cuts down on redundant computations
  - Quadtrees: efficient sparse representation
  - Neighbourhood maps: standard technique
- Experimental evaluation
  - Semi-naïve is good
  - memory/time tradeoff

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